## Ecuații de forma: $f(\sin x)=0$, $f(\cos x)=0$, $f(tg\, x)=0$

Exp: (1) $2\sin^2 x - 3\sin x + 1 = 0$.

Not $\boxed{\sin x = t}$ $\Rightarrow$ $2t^2 - 3t + 1 = 0$, $\Delta = 1$, $t_1 = \frac{1}{2}$, $t_2 = 1$.

$\sin x = \frac{1}{2} \Rightarrow x \in \{(-1)^k \frac{\pi}{6} + k\pi / k \in \mathbb{Z}\}$ ($S_1$)

$\sin x = 1 \Rightarrow x \in \{(-1)^k \frac{\pi}{2} + k\pi / k \in \mathbb{Z}\}$ ($S_2$)

$\Rightarrow S = S_1 \cup S_2$.

(2) $\cos 2x - 3\cos x = 4\cos^2 \frac{x}{2}$

(3) $tg^2 x - 3 tg\, x + 2 = 0$

Atenție! $\cos^2 x - \sin^2 x = \cos 2x$

$2\cos^2 x - 1 = \cos 2x$

$\cos^2 x = \frac{1 + \cos 2x}{2}$

$\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}$

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## Ecuații omogene în $\sin f(x)$ și $\cos f(x)$

Exp: (1) $\sin^2 x - 2\sin x \cos x - \underline{3\cos^2 x} = 0$.

Dacă $\cos^2 x = 0 \Rightarrow \sin^2 x - 2\cdot 0 - 3\cdot 0 = 0 \Rightarrow \sin x = 0$ fals. de ce?

$\left.\begin{array}{l}\sin^2 x + \cos^2 x = 1\\ \cos x = 0\end{array}\right| \Rightarrow \sin^2 x = 1.$

În concluzie $\cos x \neq 0$.

$\sin^2 x - 2\sin x \cos x - 3\cos^2 x = 0 \quad | : \cos^2 x$

$\frac{\sin^2 x}{\cos^2 x} - \frac{2\sin x \cos x}{\cos^2 x} - 3\frac{\cos^2 x}{\cos^2 x} = 0$.

$\left(\frac{\sin x}{\cos x}\right)^2 - 2\cdot\left(\frac{\sin x}{\cos x}\right) - 3 = 0$. Not $\frac{\sin x}{\cos x} = t$.

$t^2 - 2t - 3 = 0$, $\Delta = 16$, $t_1 = -1$, $t_2 = 3$.

$tg\, x = t_1 \Rightarrow tg\, x = -1 \Rightarrow x \in \{-\operatorname{arctg} 1 + k\pi / k \in \mathbb{Z}\}$

$x \in \{-\frac{\pi}{4} + k\pi / k \in \mathbb{Z}\}$. ($S_1$)

$tg\, x = t_2 \Rightarrow tg\, x = 3 \Rightarrow x \in \{\operatorname{arctg} 3 + k\pi / k \in \mathbb{Z}\}$ ($S_2$)

$S = S_1 \cup S_2$

(2) $\cos^2 x - 3\sin x \cos x = 0$

(3) $6\sin^2 x + \sin x \cos x - \cos^2 x = 2$

## Ecuații liniare $\quad a \sin f(x) + b \cos f(x) = c, \quad a, b, c \in \mathbb{R}$

Exp: (1) $\sin x + 7\cos x = 5$.

Folosim substituția universală: $\boxed{tg\frac{x}{2} = t}$ $\quad \sin x = \frac{2t}{1+t^2}, \quad \cos x = \frac{1-t^2}{1+t^2}$

$$\frac{2t}{1+t^2} + 7\cdot\frac{1-t^2}{1+t^2} = 5 \Rightarrow 2t + 7 - 7t^2 - 5 - 5t^2 = 0$$

$$\Rightarrow -12t^2 + 2t + 2 = 0 \;|:(-2) \Rightarrow 6t^2 - t - 1 = 0. \quad t_1 = \frac{1}{2}, \; t_2 = -\frac{1}{3}.$$

$tg\frac{x}{2} = \frac{1}{2} \Rightarrow x \in \{2\,arctg\frac{1}{2} + k\pi / k \in \mathbb{Z}\}$ ($S_1$)

$tg\frac{x}{2} = -\frac{1}{3} \Rightarrow x \in \{-2\,arctg\frac{1}{3} + k\pi / k \in \mathbb{Z}\}$ ($S_2$)

$\Rightarrow S = S_1 \cup S_2$

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## Ecuații simetrice de forma $\quad a[\sin f(x) + \cos f(x)] + b \sin f(x) \cos f(x) = c$

Exp: (1) $\sin 2x + 5(\sin x + \cos x) = -1$ $\quad \boxed{\sin 2x = 2\sin x \cos x}$

$$2\sin x \cos x + 5(\sin x + \cos x) = -1$$

Not $\sin x + \cos x = t \;/^2$

$$\sin^2 x + 2\sin x \cos x + \cos^2 x = t^2 \Rightarrow 2\sin x \cos x = t^2 - 1$$

Înlocuim în ec inițială: $t^2 - 1 + 5t = -1 \Rightarrow t^2 + 5t = 0 \quad t_1 = -5, \; t_2 = 0$

Verificăm dacă sol $\in [-\sqrt{2}; \sqrt{2}]$ deoarece:

$$|\sin x + \cos x| = \left|\sin x + \cos\left(\frac{\pi}{2} - x\right)\right| = \sqrt{2}\left|\cos\left(x - \frac{\pi}{2}\right)\right| \le \sqrt{2}$$

$\sin x + \cos x = -5$ fals

$\sin x + \cos x = 0 \Rightarrow tg x = -1$ etc.