# Teorema fundamentală a asemănării

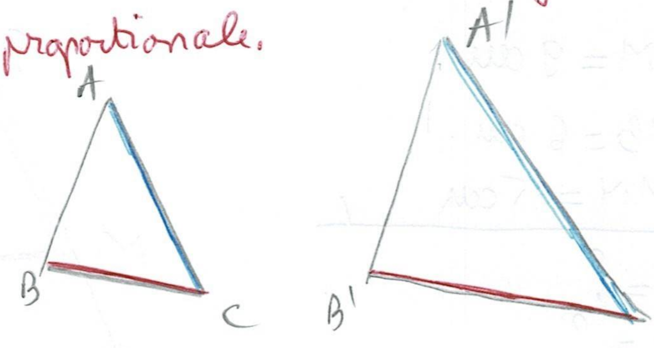
Două triunghiuri se numesc asemenea dacă au toate unghiurile congruente iar laturile respectiv proporționale.

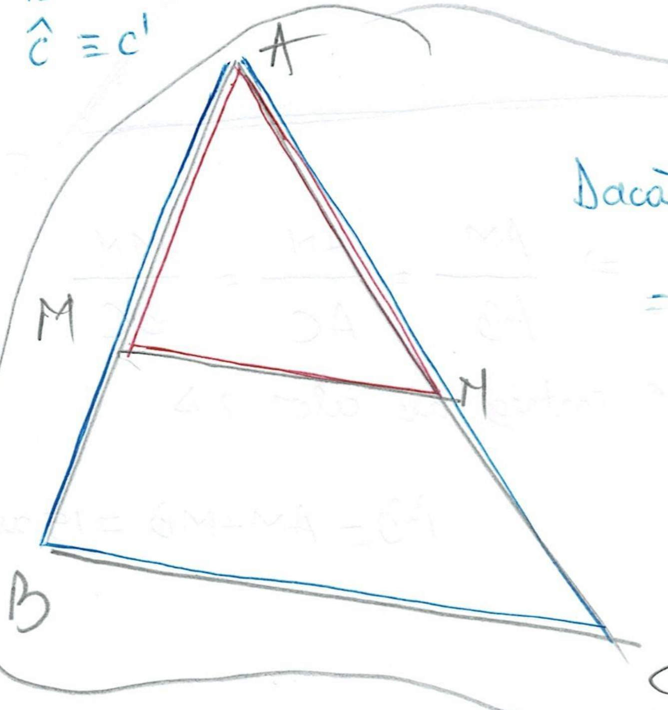
$\Delta ABC \sim \Delta A'B'C'$

$\hat{A} \equiv \hat{A'}$ $\qquad \frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'}$

$\hat{B} \equiv \hat{B'}$

$\hat{C} \equiv \hat{C'}$



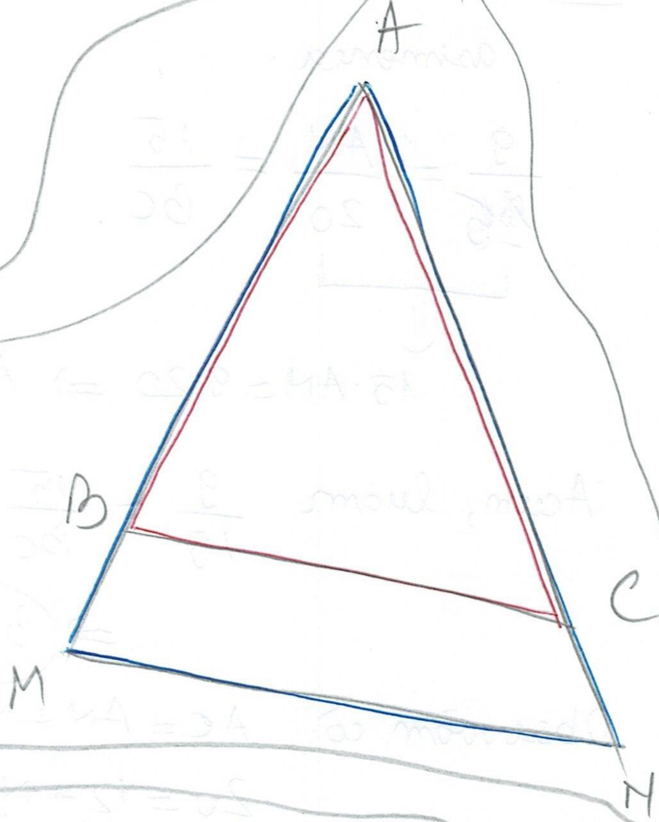



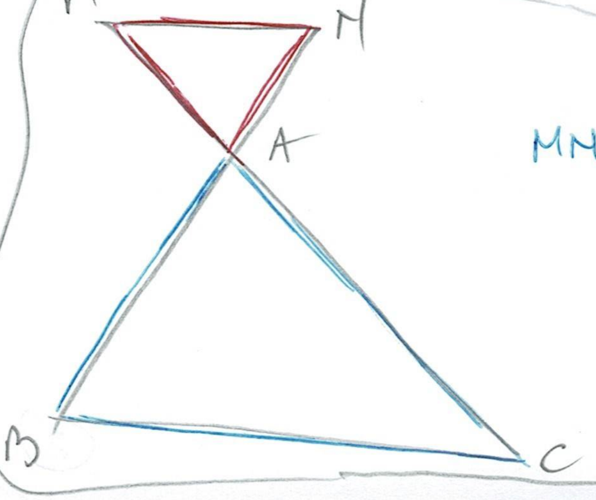

Dacă $MN \parallel BC \Rightarrow \Delta AMN \sim \Delta ABC$.

$\Rightarrow \frac{AM}{AB} = \frac{AN}{AC} = \frac{MN}{BC}$.

$MN \parallel BC \Rightarrow \Delta ABC \sim \Delta AMN$

$\Rightarrow \frac{AB}{AM} = \frac{AC}{AN} = \frac{BC}{MN}$.




$MN \parallel BC \Rightarrow \frac{AN}{AB} = \frac{AM}{AC} = \frac{MN}{BC}$

$\Delta ABC \sim \Delta AMN$

① $\Delta ABC$ oarecare

$AC = 20$ cm

$MN \parallel BC$ astfel încât

$AM = 9$ cm

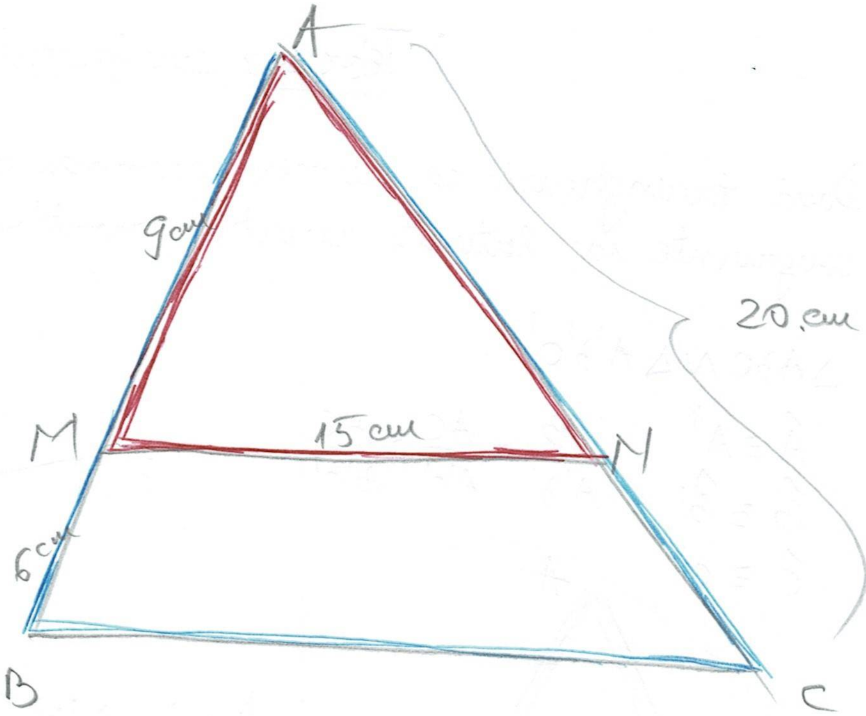
$MB = 6$ cm

$MN = 15$ cm

$AN = ?$

$NC = ?$

$BC = ?$




$$MN \parallel BC \xrightarrow{T.F.A} \Delta AMN \sim \Delta ABC \Rightarrow \frac{AM}{AB} = \frac{AN}{AC} = \frac{MN}{BC}$$

Obs: Se iau OBLIGATORIU laturile întregi ale celor 2 $\Delta$ asemenea.

$AB = AM + MB = 15$ cm.

$$\frac{9}{15} = \frac{AN}{20} = \frac{15}{BC}.$$

$$\Downarrow$$

$$15 \cdot AN = 9 \cdot 20 \Rightarrow AN = \frac{\cancel{9}^{3} \cdot \cancel{20}^{4}}{\cancel{15}_{\cancel{3}_{1}}} \Rightarrow AN = 12 \text{ cm}$$

Acum, luăm $\frac{9}{15} = \frac{15}{BC} \Rightarrow 9 \cdot BC = 15 \cdot 15 \Rightarrow BC = \frac{\cancel{15}^{5} \cdot \cancel{15}^{5}}{\cancel{9}_{\cancel{3}_{1}}}$

$\Rightarrow BC = 25$ cm

Observăm că $AC = AN + NC$

$20 = 12 + NC$

$NC = 8$ cm

$$\Downarrow$$

② $\Delta DEF$, $M \in DE$ a.î. $\frac{DM}{ME} = \frac{2}{3}$.

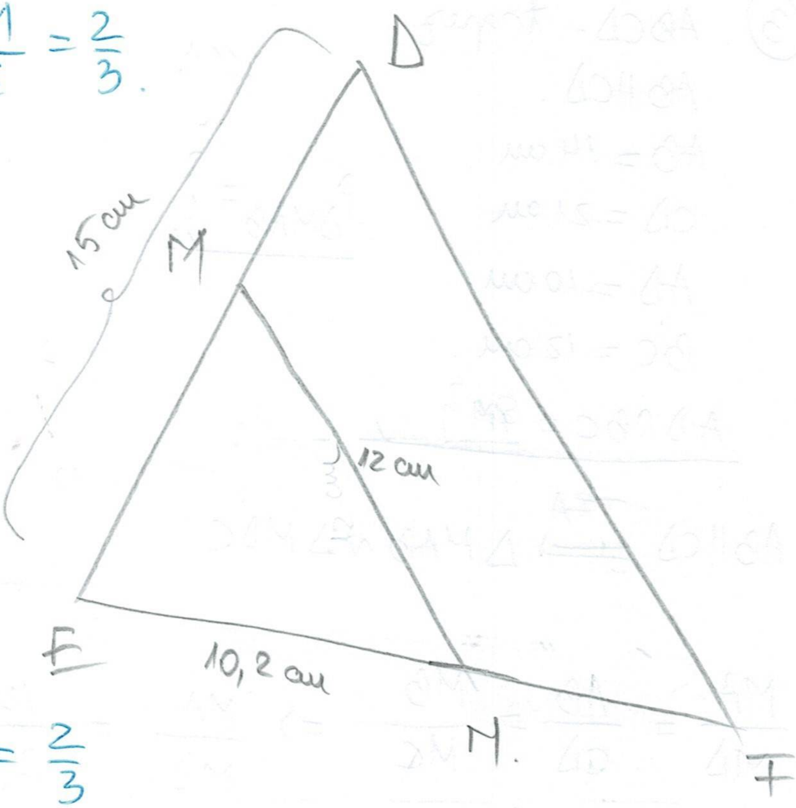
$DE = 15$ cm.

$N \in EF$ a.î. $MN \parallel DF$.

$EN = 10{,}2$ cm

$MN = 12$ cm.

$P_{\Delta DEF} = ?$




$$\left.\begin{array}{l}\frac{DM}{ME} = \frac{2}{3} \\ ME = DE - DM\end{array}\right| \Rightarrow \frac{DM}{DE-DM} = \frac{2}{3}$$

$$\frac{DM}{15-DM} = \frac{2}{3} \Rightarrow 3 \cdot DM = 2 \cdot (15 - DM)$$

$$3DM = 30 - 2DM$$

$$5DM = 30$$

$$DM = 6 \text{ cm.}$$

$ME = DE - DM = 15 - 6 = 9$ cm.

$P_{\Delta DEF} = DE + EF + FD = 15 + EF + FD$

$MN \parallel DF \xRightarrow{TFA} \Delta EMN \sim \Delta EDF \Rightarrow \frac{EM}{ED} = \frac{EN}{EF} = \frac{MN}{DF}$

$\Rightarrow \frac{9}{15} = \frac{10{,}2}{EF} = \frac{12}{DF}$

$\Downarrow$

$9EF = 15 \cdot 10{,}2$

$EF = \frac{\cancel{15}^{5} \cdot 10{,}2}{\cancel{9}_{3}}$

$EF = \frac{51}{3} = 17$ cm

$\frac{9}{15} = \frac{12}{DF}$

$9DF = 15 \cdot 12 \Rightarrow DF = \frac{\cancel{15}^{5} \cdot \cancel{12}^{4}}{\cancel{9}_{\cancel{3}}}$

$DF = 20$ cm

$P_{\Delta DEF} = 15 + 17 + 20 = 52$ cm

(3) ABCD - trapez.

$AB \parallel CD$.

$AB = 14$ cm

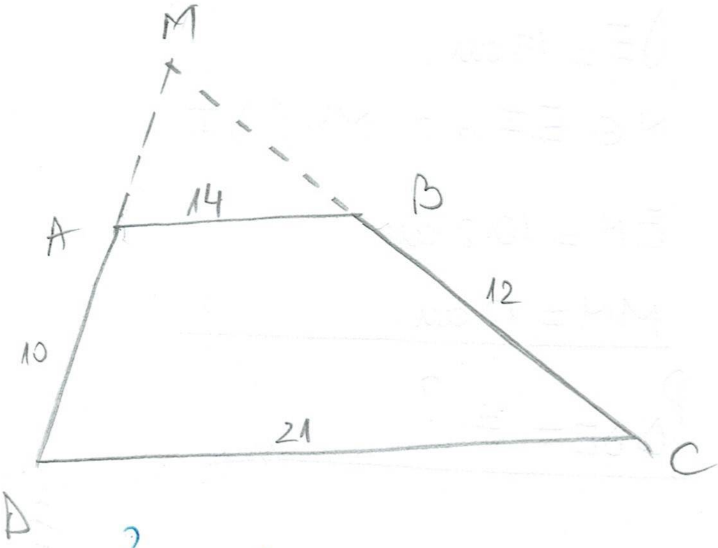
$CD = 21$ cm

$AD = 10$ cm

$BC = 12$ cm.

$AD \cap BC = \{M\}$

$P_{\Delta MAB} = ?$




$AB \parallel CD \xRightarrow{TFA} \Delta MAB \sim \Delta MDC$

$$\frac{MA}{MD} = \frac{AB}{CD} = \frac{MB}{MC} \Rightarrow \frac{MA}{MD} = \frac{\cancel{14}^{2}}{\cancel{21}_{3}} = \frac{MB}{MC}.$$

$MA + AD = MD \Rightarrow MD = MA + 10.$

$MB + BC = MC \Rightarrow MC = MB + 12.$

$$\frac{MA}{MA+10} = \frac{2}{3} = \frac{MB}{MB+12}$$

$3MA = 2(MA+10)$

$3MA = 2MA + 20.$

$MA = 10$ cm.

$3MB = 2(MB+12)$

$3MB = 2MB + 24.$

$MB = 24$ cm

$P_{\Delta MAB} = MA + AB + MB = 10 + 14 + 24 = 48$ cm.