



Model test-Numere complexe

(1) Fie $z_1 = 2i - 5$, $z_2 = 1 - i$. Calculați:

a) $3z_1 - 5z_2$

$$\begin{aligned} 3z_1 - 5z_2 &= 3(2i - 5) - 5(1 - i) \\ &= \underline{6i} - \underline{15} - \underline{5} + \underline{5i} \\ &= 11i - 20 \end{aligned}$$

b) $z_1 \cdot z_2$

$$\begin{aligned} z_1 \cdot z_2 &= (2i - 5)(1 - i) = 2i(1 - i) - 5(1 - i) = \\ &= 2i - 2i^2 - 5 + 5i = \\ &= \underline{2i} + \underline{2} - 5 + \underline{5i} = \\ &= 7i - 3 \end{aligned}$$

c) $\frac{z_1}{z_2}$

$$\frac{z_1}{z_2} = \frac{2i - 5}{1 - i} = \frac{(1 + i)(2i - 5)}{(1 + i)(1 - i)} = \frac{1(2i - 5) + i(2i - 5)}{1^2 - i^2} =$$

$$(a+b)(a-b) = a^2 - b^2$$

$$= \frac{2i - 5 + 2i^2 - 5i}{1 + 1} = \frac{2i - 5 - 2 - 5i}{2} = \frac{-3i - 7}{2}$$

d) $\overline{z_1}$

$$\overline{z_1} = \overline{2i - 5} = -2i - 5$$

e) $|z_2|$

$$|z_2| = |1 - i| = \sqrt{1^2 + (-1)^2} = \sqrt{1 + 1} = \sqrt{2}$$

(2) Fără a rezolva ecuația $2x^2 - 3x + 5 = 0$

calculati: a) $\frac{x_1}{x_2} + \frac{x_2}{x_1}$

b) $x_1^3 + x_2^3$

$$2x^2 - 3x + 5 = 0$$

$$\begin{array}{l|l} a=2 & \text{Viete} \\ b=-3 & \Rightarrow \\ c=5 & \end{array} \begin{cases} x_1 + x_2 = -\frac{b}{a} = -\frac{-3}{2} = \frac{3}{2} \\ x_1 \cdot x_2 = \frac{c}{a} = \frac{5}{2} \end{cases}$$

$$\begin{aligned} \text{a) } \frac{x_1}{x_2} + \frac{x_2}{x_1} &= \frac{x_1^2 + x_2^2}{x_1 x_2} = \frac{(x_1 + x_2)^2 - 2x_1 x_2}{x_1 x_2} \\ &= \frac{\left(\frac{3}{2}\right)^2 - 2 \cdot \frac{5}{2}}{\frac{5}{2}} = \frac{\frac{9}{4} - \frac{5}{1}}{\frac{5}{2}} = \frac{\frac{9-20}{4}}{\frac{5}{2}} = \frac{-11}{4} \cdot \frac{2}{5} = -\frac{11}{10} \end{aligned}$$

$$\begin{aligned} \text{b) } x_1^3 + x_2^3 &= (x_1 + x_2)^3 - 3x_1 x_2 (x_1 + x_2) \\ &= \left(\frac{3}{2}\right)^3 - 3 \cdot \frac{5}{2} \cdot \frac{3}{2} = \frac{27}{8} - \frac{45}{4} \\ &= \frac{27}{8} - \frac{90}{8} = -\frac{63}{8} \end{aligned}$$

③ Determinați ecuația de gradul al doilea care are o soluție $2-5i$.

$$x^2 - Sx + P = 0, \quad \begin{array}{l} S = x_1 + x_2 \\ P = x_1 \cdot x_2 \end{array}$$

$$x_1 \in \mathbb{C} \text{ nol} \Rightarrow x_2 = \overline{x_1} \text{ nol}$$

$$x_1 = 2-5i \Rightarrow x_2 = \overline{x_1} = \overline{2-5i} = 2+5i$$

$$S = x_1 + x_2 = 2-5i + 2+5i = 4$$

$$P = x_1 \cdot x_2 = (2-5i)(2+5i) = 2^2 - (5i)^2 = 4 - 25i^2 = 4 + 25 = 29$$

$$x^2 - 4x + 29 = 0$$

④ Scrieți ecuația de gradul al doilea în y care are soluțiile $y_1 = \frac{1}{x_1}, y_2 = \frac{1}{x_2}$ unde $x_1 \neq x_2$

nicht lösbar, da $x^2 - x + 2 = 0$

$$y^2 - Sy + P = 0, \quad S = y_1 + y_2$$

$$P = y_1 \cdot y_2$$

$$S = y_1 + y_2 = \frac{x_2}{\frac{1}{x_1}} + \frac{x_1}{\frac{1}{x_2}} = \frac{x_2 + x_1}{\frac{1}{x_1 x_2}}$$

$$P = y_1 \cdot y_2 = \frac{1}{x_1} \cdot \frac{1}{x_2} = \frac{1}{x_1 x_2}$$

$$x^2 - x + 2 = 0$$

$$\begin{array}{l|l} a=1 & \\ b=-1 & \Rightarrow x_1 + x_2 = -\frac{b}{a} = -\frac{-1}{1} = 1 \\ c=2 & \end{array} \quad \begin{array}{l} x_1 \cdot x_2 = \frac{c}{a} = \frac{2}{1} = 2 \end{array}$$

$$\begin{array}{l|l} S = y_1 + y_2 = \frac{1}{2} & \\ P = y_1 \cdot y_2 = \frac{1}{2} & \end{array} \Rightarrow \begin{array}{l} y^2 - \frac{1}{2}y + \frac{1}{2} = 0 \\ 2y^2 - y + 1 = 0 \end{array}$$