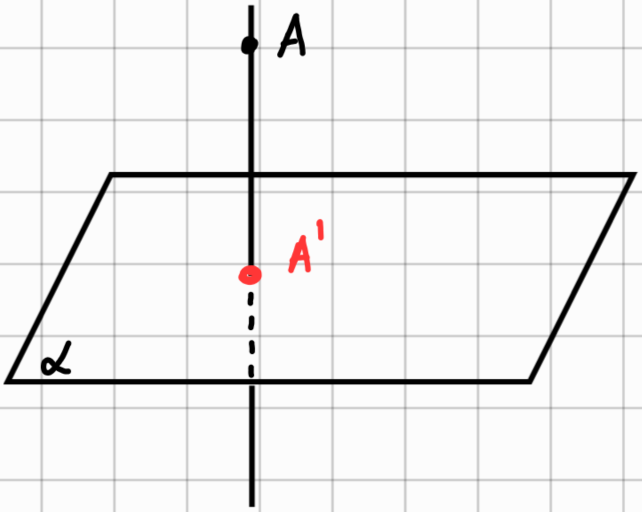
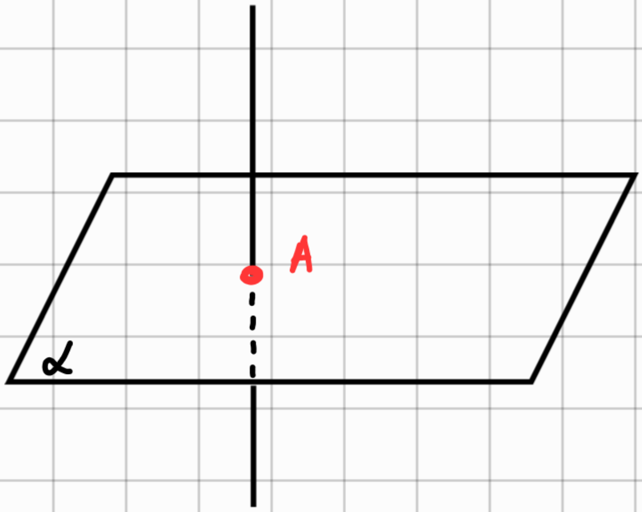
PROIECȚII ORTOGONALE

① Proiecția ortogonală a unui punct pe un plan.



$A' = pr_{\alpha} A$



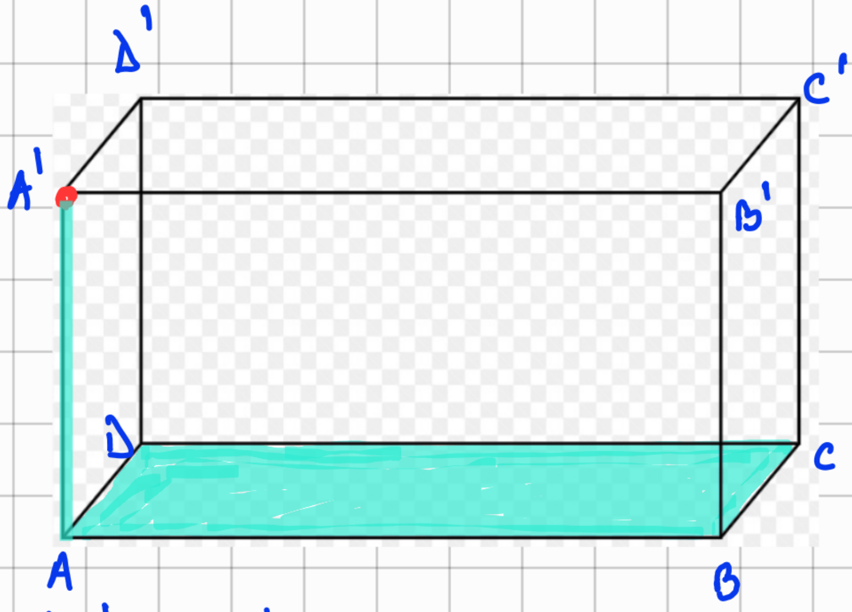
$A = pr_{\alpha} A$

Exp rezolvat

① $ABCDA'B'C'D'$ – paralelip. dreptunghic

a) $pr_{(ABC)} A' = ?$

b) $pr_{(BCC')} A = ?$



a) $A'ABB'$ – dreptunghi $\Rightarrow AA' \perp AB$

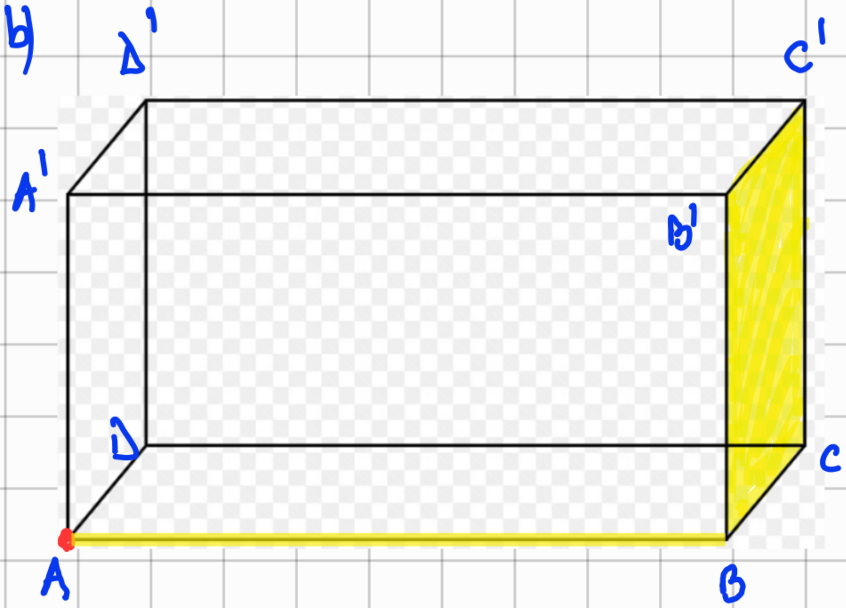
$A'ADD'$ – dreptunghi $\Rightarrow AA' \perp AD$

$AB, AD \subset (ABC)$

$\Rightarrow AA' \perp (ABC) \Rightarrow$

$\Rightarrow pr_{(ABC)} A' = A.$

b)



ABCD dreptunghi

$\Downarrow$

$AB \perp BC$

$ABB'A'$ dreptunghi

$\Downarrow$

$AB \perp BB'$

$$\left.\begin{array}{l} AB \perp BC \\ AB \perp BB' \\ BC, BB' \subset (BCC') \end{array}\right| \Rightarrow AB \perp (BCC') \Rightarrow pr_{(BCC')} A = B$$

Ex propus

a) $pr_{(BB'C')} A' = ?$

b) $pr_{(ADC)} C' = ?$

c) $pr_{(DCC')} D' = ?$

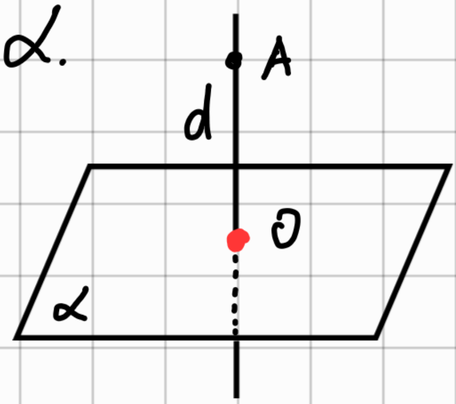
d) $pr_{(ADD')} C = ?$

2 Proiecția ortogonală a unei drepte pe un plan

Fie dreapta d și un plan α.

Cazul 1: $\boxed{d \perp \alpha}$

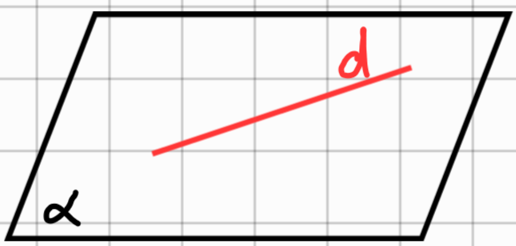
$d \cap \alpha = \{O\}$



$$\text{pr}_\alpha A = O \Rightarrow \boxed{\text{pr}_\alpha d = \{O\}}$$

Caz 2: $\boxed{d \not\perp \alpha}$

a) $d \subset \alpha \Rightarrow \boxed{\text{pr}_\alpha d = d.}$



b) $d \not\subset \alpha$

Există $A, B \in d$, $A, B \notin \alpha$

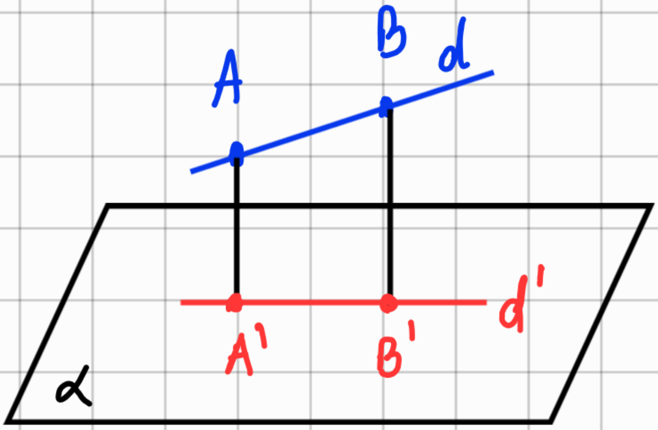
$\left.\begin{array}{l} \text{pr}_\alpha A = A' \Rightarrow AA' \perp d \\ \text{pr}_\alpha B = B' \Rightarrow BB' \perp d \end{array}\right| \Rightarrow$

$\Rightarrow AA' \parallel BB' \Rightarrow$

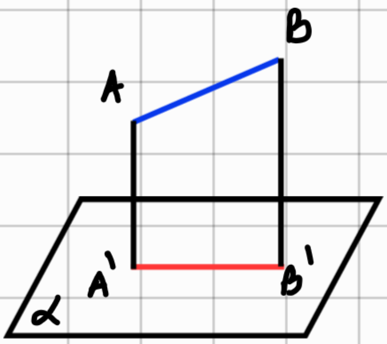
$\Rightarrow \alpha \cap (AA'B') = A'B' \Rightarrow \text{pr}_\alpha AB = A'B'$

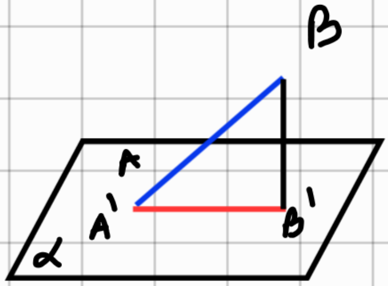
$\Downarrow$

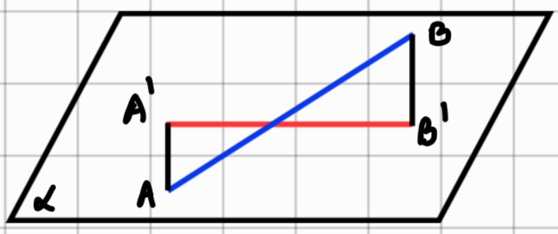
$\boxed{\text{pr}_\alpha d = d'}$



Obs: Pentru a determina proiecția unui segment AB pe un plan α proiectăm capetele segmentului pe planul α.







Exemplu rezolvat

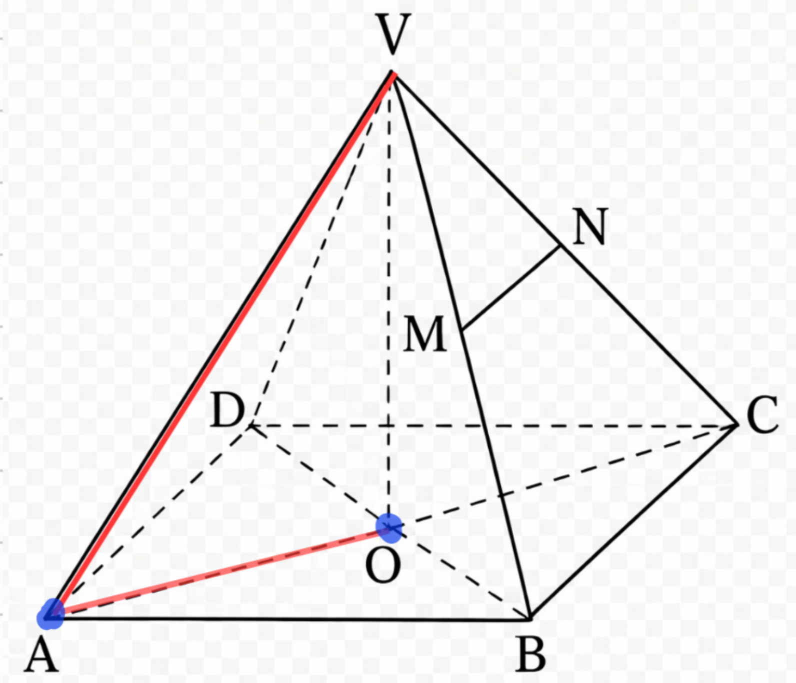
VABCD – piramidă patrulateră regulată

M – mij VB, N – mij VC, $AC \cap BD = \{O\}$

a) $\text{pr}_{(ABCD)} VA = ?$

b) $pr_{(ABCD)} VM = ?$

c) $pr_{(ABCD)} ON = ?$



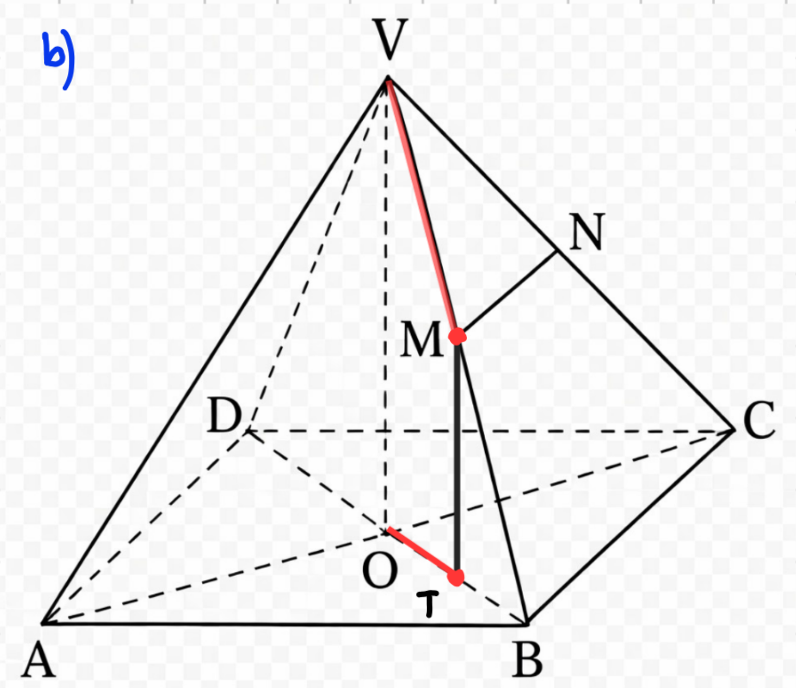
a) ABCD − pătrat

$VO \perp (ABCD)$

$$\left.\begin{array}{l} A \in (ABCD) \Rightarrow pr_{(ABCD)} A = A \\ VO \perp (ABCD) \Rightarrow pr_{(ABCD)} V = O \end{array}\right| \Rightarrow$$

$\Rightarrow pr_{(ABCD)} AV = AO.$

b)



$pr_{(ABCD)} VM = ?$

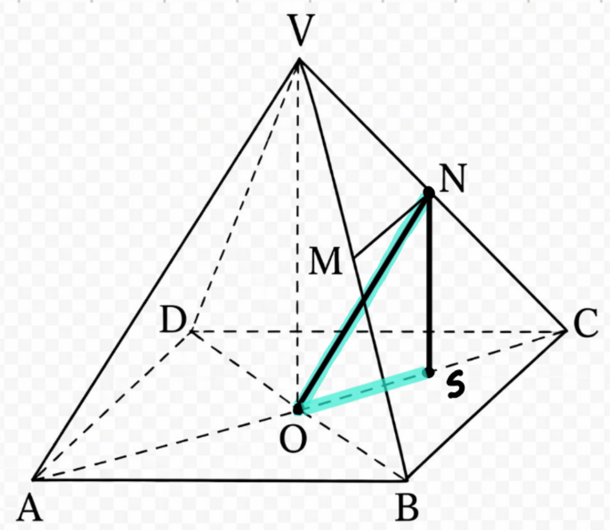
$$\left.\begin{array}{l} \text{Fie } T - \text{mij. } OB \\ M - \text{mij } VB \\ pr_{(ABCD)} VB = OB \end{array}\right| \Rightarrow$$

$$\left.\begin{array}{l} \Rightarrow pr_{(ABCD)} M = T \\ pr_{(ABCD)} V = O \end{array}\right| \Rightarrow$$

$\Rightarrow pr_{(ABCD)} VM = OT$

c) $pr_{(ABCD)} ON = ?$

$$\left.\begin{array}{l} \text{Fie } S - \text{mij } OC \\ N - \text{mij } VC \\ pr_{(ABCD)} VC = OC \end{array}\right| \Rightarrow pr_{(ABCD)} N = S$$



$$\left.\begin{array}{l} pr_{(ABCD)} M = S \\ pr_{(ABCD)} O = O \end{array}\right| \Rightarrow pr_{(ABCD)} OM = OS$$

Exemplu propus

a) $pr_{(ABCD)} VC = ?$

b) $pr_{(ABCD)} MN = ?$

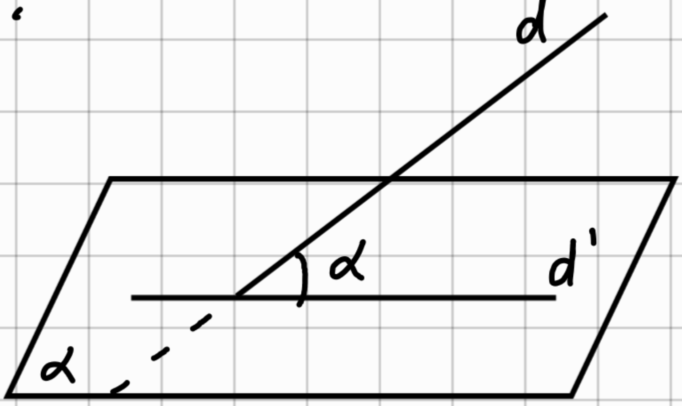
c) $pr_{(ABCD)} VN = ?$

d) $pr_{(ABCD)} OM = ?$

3 Unghiul dintre o dreaptă și un plan.

$$\boxed{\sphericalangle(d,\alpha) = \sphericalangle(d,d')}$$

$d' = pr_{\alpha} d.$



Unghiul dintre o dreaptă d și un plan α $(d \not\perp \alpha)$ este unghiul dintre dreapta d și proiecția ei pe plan (d').

Exemple rezolvate:

1. $ABCDA'B'C'D'$ – cub.

a) $\sphericalangle(AA', B'C'D') = ?$

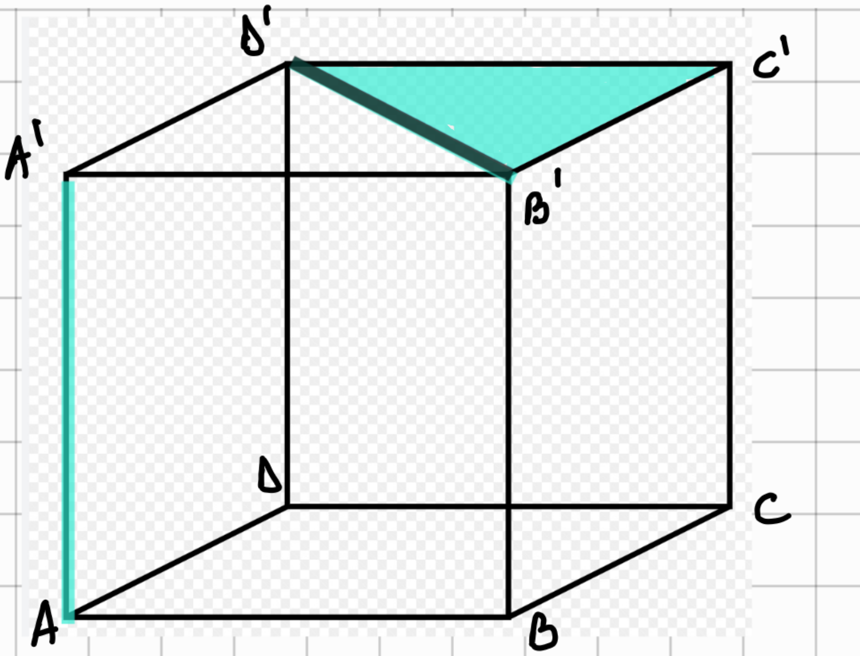
b) $\sphericalangle(BC', ACD) = ?$

c) $\sphericalangle(A'C', ABC) = ?$

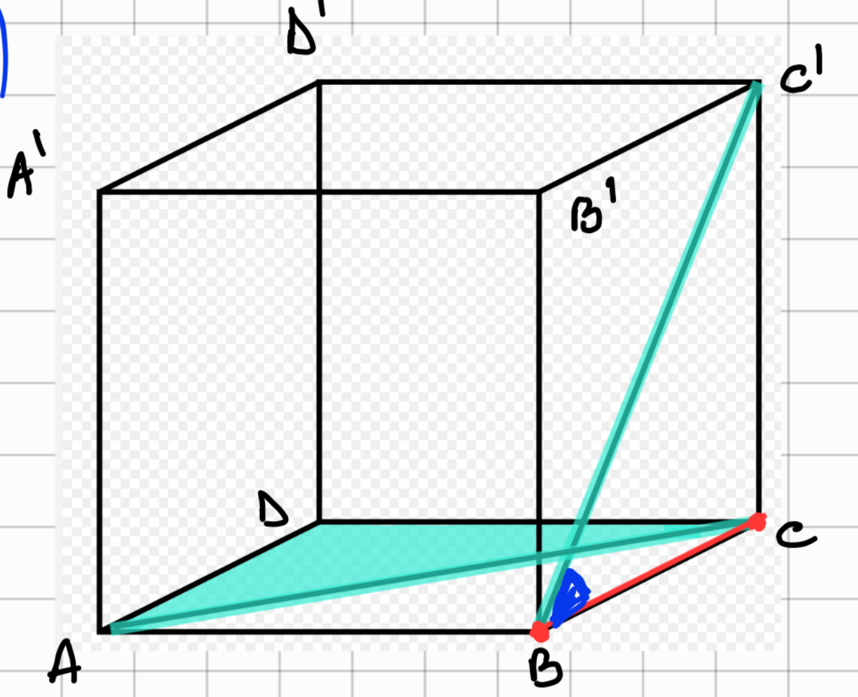
a) $\left.\begin{array}{l} AA' \perp A'B' \\ AA' \perp A'D' \\ A'B', A'D' \subset (B'C'D') \\ A'B' \cap A'D' = \{A'\} \end{array}\right| \Rightarrow$

$\Rightarrow AA' \perp (B'C'D') \Rightarrow$

$\sphericalangle(AA', (B'C'D')) = 90^\circ$



b)



$\left.\begin{array}{l} pr_{(ACD)}\, C' = C \\ pr_{(ACD)}\, B = B \end{array}\right| \Rightarrow$

$\Rightarrow pr_{(ACD)}\, BC' = BC$

$\Downarrow$

$\sphericalangle(BC', ACD) = \sphericalangle(BC', BC)$

$= 45^\circ$

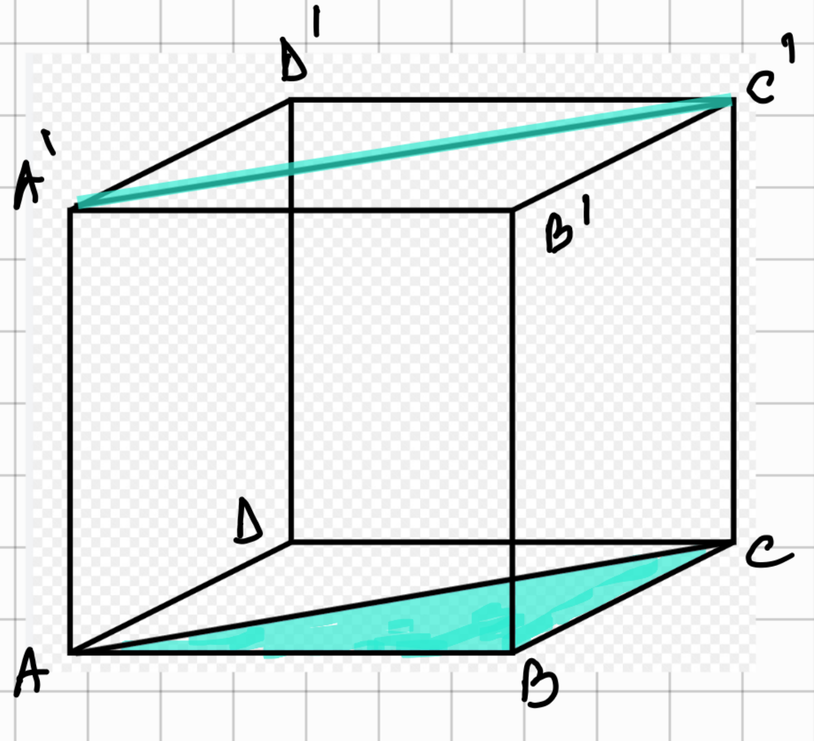
c) $\left.\begin{array}{l} ADD'A' \text{ - pătrat} \Rightarrow \\ \Rightarrow AA' = DD',\ AA' \parallel DD' \\ DCC'D' \text{ - pătrat} \Rightarrow \\ \Rightarrow DD' = CC',\ DD' \parallel CC' \end{array}\right| \Rightarrow$

$\Rightarrow AA' = CC',\ AA' \parallel CC' \Rightarrow$

$\Rightarrow ACC'A'$ - paralelogram

$\left.\begin{array}{l} \Rightarrow A'C' \parallel AC \\ AC \subset (ABC) \end{array}\right| \Rightarrow A'C' \parallel (ABC)$

$\Downarrow$



$\Rightarrow \measuredangle(A'C', ABC) = \measuredangle(AC, ABC) = 0^\circ.$