S 1

Să se determine mulțimea valorilor parametrului real m pentru care matricea $\begin{pmatrix} 2 & x & 3 \\ m & x-1 & 1 \\ 1 & 1 & x \end{pmatrix}$ are rangul trei pentru orice $x < 0$. **(9 pct.)**

a) $\left(\frac{2}{3}, 1\right)$; b) $\left[\frac{5}{3}, 2\right]$; c) $\left(\frac{1}{2}, 2\right)$; d) $\left[-1, \frac{4}{3}\right] \cup \{2\}$; e) $\left[-\frac{1}{3}, 2\right]$; f) $\left(\frac{1}{3}, 1\right)$.

Fie $A = \begin{pmatrix} 2 & x & 3 \\ m & x-1 & 1 \\ 1 & 1 & x \end{pmatrix}$; $r = 3 \Leftrightarrow \det A \neq 0$

$$\det A = \begin{vmatrix} 2 & x & 3 \\ m & x-1 & 1 \\ 1 & 1 & x \end{vmatrix} = 2(x^2-x) + 3m + x - 3(x-1) - 2 - mx^2 =$$
$$= 2x^2 - 2x + 3m + x - 3x + 3 - 2 - mx^2 =$$
$$= (2-m)x^2 - 4x + (3m+1)$$

Fie $f: \mathbb{R} \to \mathbb{R}$, $f(x) = (2-m)x^2 - 4x + (3m+1)$

$\underline{f(x) \neq 0, \ \forall x < 0}$

$\Delta = 16 - 4(2-m)(3m+1) = 4(3m^2 - 5m + 2)$

Caz 1: $\boxed{\Delta < 0} \Rightarrow 3m^2 - 5m + 2 < 0 \Rightarrow \boxed{m \in \left(\frac{2}{3}; 1\right)}$

$\Rightarrow f(x) \neq 0, \ \forall x < 0$

Caz 2: $\boxed{\Delta = 0} \Rightarrow m_1 = \frac{2}{3}, \ m_2 = 1$

$m = 1 \Rightarrow f(x) = x^2 - 4x + 4 = (x-2)^2 \geq 0$

$f(x) = 0$ pt $x = 2 > 0$ $\Big| \Rightarrow f(x) \neq 0 \ \forall x < 0$

$\Rightarrow \boxed{m = 1}$ convine

$m = \frac{2}{3} \Rightarrow f(x) = \frac{4}{3}x^2 - 4x + 3 = \frac{1}{3}(2x-3)^2 \geq 0$ ~~$\Rightarrow f(x) \neq 0 \ \forall x < 0$~~

$f(x) = 0$ pt $x = \frac{3}{2} > 0$

$\boxed{m = \frac{2}{3}}$ convine.

Caz 3: $\boxed{\Delta > 0} \Rightarrow m \in \left(-\infty; \frac{2}{3}\right) \cup (1; +\infty)$

$\Delta > 0 \Rightarrow x_1, x_2 \in \mathbb{R}$

Punem condiția ca $x_1, x_2 \geq 0 \Rightarrow \begin{cases} S > 0 \\ P > 0 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} \frac{4}{2-m} > 0 \\ \frac{3m+1}{2-m} > 0 \end{cases}$

Dacă $2 - m > 0 \Rightarrow m < 2 \Rightarrow \begin{cases} \frac{4}{2-m} > 0 \quad \text{(A)} \\ \frac{3m+1}{2-m} > 0 \Rightarrow m > -\frac{1}{3} \end{cases}$

Dacă $2 - m < 0 \Rightarrow m > 2 \Rightarrow \begin{cases} \frac{4}{2-m} < 0 \quad \text{(F)} \end{cases}$

$\Rightarrow \boxed{m \in \left(-\frac{1}{3}, 2\right)}$

Pt $m = 2 \Rightarrow f(x) = -4\underbrace{x}_{<0} + 7 \Rightarrow f(x) > 0 \Rightarrow \boxed{m = 2}$

Pt $m = -\frac{1}{3} \Rightarrow f(x) = \frac{7}{3}x^2 - 4x = x\left(\frac{7x}{3} - 4\right)$

$\Rightarrow x = 0$ sau $x = \frac{12}{7} > 0 \Rightarrow \boxed{m = -\frac{1}{3}}$

Concluzie: $m \in \left[-\frac{1}{3}, 2\right]$