



PREADMITERE POLITEHNICĂ 2026

7. Fie z un număr complex, $z \in \mathbb{C} \setminus \mathbb{R}$. Dacă $\frac{z}{z^2+4}$ este număr real, să se calculeze $|z|$. (9 pct.)

a) $|z| = 4$; b) $|z| = \frac{\sqrt{3}}{2}$; c) $|z| = 2\sqrt{2}$; d) $|z| = \sqrt{3}$; e) $|z| = \frac{1}{2}$; f) $|z| = 2$.

met1: $z = a + bi$

$$\frac{z}{z^2+4} = \frac{a+bi}{a^2+2abi-b^2+4} = \frac{(a^2-b^2+4)-2abi}{(a^2-b^2+4)+2abi} =$$

$$= \frac{a^3-ba^2+4a+\widehat{a^2bi}-\widehat{b^3i}+\widehat{4bi}-\widehat{2a^2bi}+\widehat{2ab^2}}{(a^2-b^2+4)^2+4a^2b^2}$$

$$= \frac{(a^3-ba^2+2ab^2+4a) - i(b^3+a^2b-4b)}{(a^2-b^2+4)^2+4a^2b^2}$$

$$\Rightarrow b^3+a^2b-4b=0 \Rightarrow b(b^2+a^2-4)=0.$$
$$\Rightarrow b \neq 0 \text{ sau } b^2+a^2=4$$

$$|z| = \sqrt{a^2+b^2} = \sqrt{4} = 2$$

met2: $\frac{z}{z^2+4} \in \mathbb{R} \Leftrightarrow \frac{z}{z^2+4} = \overline{\left(\frac{z}{z^2+4}\right)} \Leftrightarrow$

$$\Leftrightarrow z \cdot (\bar{z}^2+4) = \bar{z} (z^2+4) \Leftrightarrow z \cdot \bar{z}^2 + 4z = \bar{z} \cdot z^2 + 4\bar{z}$$

$$\Leftrightarrow z \cdot \bar{z} (\bar{z} - z) - 4(\bar{z} - z) = 0 \Leftrightarrow (\bar{z} - z)(z\bar{z} - 4) = 0$$

$$\Rightarrow \bar{z} = z \text{ sau } z \cdot \bar{z} = 4$$

$$\underline{z = \bar{z}} \Rightarrow z \in \mathbb{R} \text{ fals } (z \in \mathbb{C} \setminus \mathbb{R})$$

$$\underline{z \cdot \bar{z} = 4} \quad \left| \Rightarrow |z|^2 = 4 \Rightarrow |z| = 2 \right.$$