C_i^o f(u)

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SIMULARE BAC 2026

TEHNOLOGIC

Subiectul II

1. $M(a) = \begin{pmatrix} 2a & a \\ a & a-2 \end{pmatrix}, a \in \mathbb{R}$

a) $\boxed{\det(M(4)) = 0}$

$M(4) = \begin{pmatrix} 8 & 4 \\ 4 & 2 \end{pmatrix} \Rightarrow \det M(4) = \begin{vmatrix} 8 & 4 \\ 4 & 2 \end{vmatrix} = 8 \cdot 2 - 4 \cdot 4 =$
$= 16 - 16 = 0$

b) $\boxed{\text{Arătați că } M(1) \cdot M(1) - M(1) = 3I_2}$

$M(1) = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix} \Rightarrow M(1) \cdot M(1) = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix} =$

$= \begin{pmatrix} 4+1 & 2-1 \\ 2-1 & 1+1 \end{pmatrix} = \begin{pmatrix} 5 & 1 \\ 1 & 2 \end{pmatrix}$

$M(1) \cdot M(1) - M(1) = \begin{pmatrix} 5 & 1 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} = 3 \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{I_2} \Rightarrow$

$\Rightarrow M(1) \cdot M(1) - M(1) = 3I_2$

c) $\boxed{\text{Dem. că } N = \det\left(M(n) - 3\left(M(1)\right)^{-1}\right) \text{ este natural impar, pentru orice nr natural nenul } n.}$

$\det(M(1)) = \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -2 - 1 = -3 \neq 0 \Rightarrow M(1)$ este matrice inversabilă

$$M(1)^t = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}, \quad M^*(1) = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$\left.\begin{aligned} a_{11} &= (-1)^{1+1} \cdot (-1) = -1 \\ a_{12} &= (-1)^{1+2} \cdot 1 = -1 \\ a_{21} &= (-1)^{2+1} \cdot 1 = -1 \\ a_{22} &= (-1)^{2+2} \cdot 2 = 2 \end{aligned}\right| \Rightarrow M^*(1) = \begin{pmatrix} -1 & -1 \\ -1 & 2 \end{pmatrix}$$

$$M^{-1}(1) = \frac{1}{\det M(1)} \cdot M^*(1) \Rightarrow$$

$$\Rightarrow M^{-1}(1) = \frac{1}{-3} \cdot \begin{pmatrix} -1 & -1 \\ -1 & 2 \end{pmatrix}$$

$$M(m) - 3 \cdot M^{-1}(1) = \begin{pmatrix} 2m & m \\ m & m-2 \end{pmatrix} - \cancel{3} \cdot \frac{1}{-\cancel{3}} \cdot \begin{pmatrix} -1 & -1 \\ -1 & 2 \end{pmatrix} =$$

$$= \begin{pmatrix} 2m & m \\ m & m-2 \end{pmatrix} + \begin{pmatrix} -1 & -1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 2m-1 & m-1 \\ m-1 & m \end{pmatrix}$$

$$\begin{vmatrix} 2m-1 & m-1 \\ m-1 & m \end{vmatrix} = (2m-1) \cdot m - (m-1) \cdot (m-1) =$$

$$= 2m^2 - m - (m^2 - 2m + 1) =$$

$$= 2m^2 - m - m^2 + 2m - 1 = m^2 + m - 1$$

$$= \underbrace{m(m+1)}_{\vdots 2} - 1 \Rightarrow \text{nr natural impar}$$

② Pe $\mathbb{R}$, $x * y = xy - 3x - 3y + 12$

a) $\boxed{2 * 5 = 1}$

$$2 * 5 = 2 \cdot 5 - 3 \cdot 2 - 3 \cdot 5 + 12 = 10 - 6 - 15 + 12 = 4 - 3 = 1.$$

b) $\boxed{\text{Arătați că } e=4 \text{ este el. neutru al legii } „*"}$

e.n.: $\exists e \in \mathbb{R}$ a.î. $e * x = x * e = x, \forall x \in \mathbb{R}$.

$e * x = x$

$ex - 3e - 3x + 12 = x$

$ex - 3e = x + 3x - 12$

$ex - 3e = 4x - 12$

$e(x-3) = 4(x-3)$

$e = 4$

$x * e = x$

$xe - 3x - 3e + 12 = x$

$xe - 3e = x + 3x - 12$

$xe - 3e = 4x - 12$

$e(x-3) = 4(x-3)$

$e = 4$

c) $\boxed{a = ?,\ a \in \mathbb{R} \text{ a.î. } (a-x)*(a-x) = (a+x)*(a+x),\ \forall x \in \mathbb{R}}$

$(a-x)*(a-x) = (a-x)(a-x) - 3(a-x) - 3(a-x) + 12$

$= a^2 - 2ax + x^2 - 6a + 6x + 12$ (1)

$(a+x)*(a+x) = (a+x)(a+x) - 3(a+x) - 3(a+x) + 12$

$= a^2 + 2ax + x^2 - 6a - 6x + 12$ (2)

Egalăm rel. (1) și (2)

$\cancel{a^2} - 2ax + \cancel{x^2} - \cancel{6a} + 6x + \cancel{12} = \cancel{a^2} + 2ax + \cancel{x^2} - \cancel{6a} - 6x + \cancel{12}$

$-2ax + 6x = 2ax - 6x$

$4ax - 12x = 0$

$4a\underbrace{(x-3)}_{\neq 0,\ \forall x \in \mathbb{R}} = 0 \Rightarrow \boxed{a = 0}$