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SIMULARE BAC 2026

ST- MAT

1. Se consideră matricea $A(a)=\begin{pmatrix}1 & a\\ -a & 1-a\end{pmatrix}$, unde a este număr real.

a) Arătați că $\det\left(A(1)\right)=1$.

b) Arătați că $A(a+b)-A(a)\cdot A(b)=abA(1)$, pentru orice numere reale a și b.

c) Determinați numerele reale a pentru care $2A(2a)-A(a)\cdot A(a)-A(1)\cdot A(2a-1)=2A(1)$.

a) $\det(A(1))=\begin{vmatrix}1 & 1\\ -1 & 0\end{vmatrix}=0+1=1$

b) $M_s=\begin{pmatrix}1 & a+b\\ -a-b & 1-a-b\end{pmatrix}-\begin{pmatrix}1 & a\\ -a & 1-a\end{pmatrix}\cdot\begin{pmatrix}1 & b\\ -b & 1-b\end{pmatrix}=$

$=\begin{pmatrix}1 & a+b\\ -a-b & 1-a-b\end{pmatrix}-\begin{pmatrix}1-ab & b+a-ab\\ -a-b+ab & -\cancel{ab}+1-a-b+\cancel{ab}\end{pmatrix}=$

$=\begin{pmatrix}ab & ab\\ -ab & 0\end{pmatrix}$

$M_d=ab\cdot\begin{pmatrix}1 & 1\\ -1 & 0\end{pmatrix}=\begin{pmatrix}ab & ab\\ -ab & 0\end{pmatrix}$

$\Rightarrow M_s=M_d,\ \forall a,b\in\mathbb{R}$

c) $A(2a)=\begin{pmatrix}1 & 2a\\ -2a & 1-2a\end{pmatrix}$, $A(2a-1)=\begin{pmatrix}1 & 2a-1\\ -2a+1 & 2-2a\end{pmatrix}$

$A(a)\cdot A(a)=\begin{pmatrix}1-a^2 & 2a-a^2\\ -2a+a^2 & 1-2a\end{pmatrix}$

$A(1)\cdot A(2a-1)=\begin{pmatrix}1 & 1\\ -1 & 0\end{pmatrix}\cdot\begin{pmatrix}1 & 2a-1\\ -2a+1 & 2-2a\end{pmatrix}=\begin{pmatrix}2-2a & 1\\ -1 & 1-2a\end{pmatrix}$

$$M_s = \begin{pmatrix} 2 & 4a \\ -4a & 2-4a \end{pmatrix} - \begin{pmatrix} 1-a^2 & 2a-a^2 \\ -2a+a^2 & 1-2a \end{pmatrix} - \begin{pmatrix} 2-2a & 1 \\ -1 & 1-2a \end{pmatrix} =$$

$$= \begin{pmatrix} a^2+2a-1 & a^2+2a-1 \\ -a^2-2a+1 & 0 \end{pmatrix}, \quad M_d = \begin{pmatrix} 2 & 2 \\ -2 & 0 \end{pmatrix}$$

$$M_s = M_d \Rightarrow a^2+2a-1 = 2 \Rightarrow a^2+2a-3=0$$

$$\Delta = 16, \quad a_1 = \frac{-2-4}{2} = -3, \quad a_2 = \frac{-2+4}{2} = 1 \quad a_1, a_2 \in \mathbb{R}$$