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SIMULARE BAC 2026

TEHNOLOGIC

Subiectul III

1. $f:(0;+\infty)\to\mathbb{R},\quad f(x)=\dfrac{x^2-3}{x^3}$

a) $\boxed{\text{Arătați că } f'(x)=\dfrac{9-x^2}{x^4},\ x\in(0,+\infty)}$

$$f'(x)=\left(\frac{x^2-3}{x^3}\right)'=\frac{(x^2-3)'\cdot x^3-(x^2-3)\cdot(x^3)'}{x^6}=$$

$$=\frac{2x\cdot x^3-(x^2-3)\cdot 3x^2}{x^6}=\frac{2x^4-3x^4+9x^2}{x^6}=$$

$$=\frac{-x^4+9x^2}{x^6}=\frac{\cancel{x^2}(-x^2+9)}{x^{\cancel{6}\,4}}=\frac{9-x^2}{x^4}$$

b) $\boxed{\text{Det. ec. tg la } G_f \text{ în pct de abscisă } x=1, \text{ situat pe graficul funcției } f}$

$(tg):\ y-f(1)=f'(1)(x-1)$

$f(1)=\dfrac{1^2-3}{1^3}=\dfrac{-2}{1}=-2$

$f'(1)=\dfrac{9-1^2}{1^4}=\dfrac{8}{1}=8$

$\Rightarrow (tg):\ y+2=8(x-1)$

$(tg):\ y+2=8x-8$

$(tg):\ -8x+y+10=0.$

c) $\boxed{\text{Dem. că } 9f(2x) - f(x) \le 4, \quad \forall x \in [1,2]}$

Fie $g : [1,2] \to \mathbb{R}$, $g(x) = 9f(2x) - f(x)$

Dem. că $\underline{g(x) \le 4}$ //

$$g'(x) = 9f'(2x) - f'(x)$$

$$f(2x) = \frac{(2x)^2 - 3}{(2x)^3} = \frac{4x^2-3}{8x^3} \Rightarrow f'(2x) = \left(\frac{4x^2-3}{8x^3}\right)' =$$

$$= \frac{(4x^2-3)' \cdot 8x^3 - (4x^2-3)\cdot(8x^3)'}{64x^6} =$$

$$= \frac{8x \cdot 8x^3 - (4x^2-3)\cdot 24x^2}{64x^6} = \frac{64x^4 - 96x^4 + 72x^2}{64x^6} =$$

$$= \frac{-32x^4 + 72x^2}{64x^6} = \frac{-\cancel{8x^2}(4x^2-9)}{8\,\cancel{64}x^{\cancel{6}\,4}} =$$

$$= \frac{9-4x^2}{8x^4}$$

$$g'(x) = 9\cdot\frac{9-4x^2}{8x^4} - {}^{8)}\frac{9-x^2}{x^4} = \frac{81 - 36x^2 - 72 + 8x^2}{8x^4} =$$

$$= \frac{9 - 28x^2}{8x^4}$$

$x \in [1,2] \Rightarrow x^2 \ge 1 \;\; / \cdot 28 \Rightarrow 28x^2 \ge 28 \;\; / (-1)$

$\Rightarrow -28x^2 \le -28 \;\; / +9 \Rightarrow 9 - 28x^2 \le -19 < 0.$

$$\Rightarrow \frac{9-28x^2}{8x^4} < 0 \Rightarrow g'(x) < 0 \Rightarrow$$

g s.d pe $[1,2]$

$$\left.\begin{array}{l} 1 \le x \le 2 \\ g \text{ s.d} \end{array}\right| \Rightarrow \underbrace{g(1)}_{\frac{25}{8}} \ge g(x) \ge \underbrace{g(2)}_{\frac{109}{64}}$$

$$\Rightarrow g(x) \le \frac{25}{8} \le 4 \Rightarrow 9f(2x) - f(x) \le 4$$

$\forall x \in [1,2]$

(2) $f : (-3; +\infty) \to \mathbb{R}$, $f(x) = x+2+\dfrac{2}{\sqrt{x+3}}$

a) $\boxed{\text{Arătați că } \int_0^4 \left(f(x) - \frac{2}{\sqrt{x+3}}\right) dx = 16}$

$$\int_0^4 \left(x+2+\cancel{\frac{2}{\sqrt{x+3}}} - \cancel{\frac{2}{\sqrt{x+3}}}\right) dx = \int_0^4 (x+2)\, dx =$$

$$= \int_0^4 x\, dx + 2\int_0^4 dx = \frac{x^2}{2}\Big|_0^4 + 2x\Big|_0^4 = \frac{4^2}{2} + 2 \cdot 4 =$$

$$= 8 + 8 = 16$$

b) $\boxed{\text{Arătați că } \int_1^6 (f(x) - x - 2)\, dx = 4}$

$$\int_1^6 \left(\cancel{x}+\cancel{2} + \frac{2}{\sqrt{x+3}} - \cancel{x} - \cancel{2}\right) dx = 2\int_1^6 \frac{1}{\sqrt{x+3}}\, dx$$

$\to 2\sqrt{t}$

Not $\sqrt{x+3} = t \Big|'$

$\frac{1}{2\sqrt{x+3}}\, dx = dt$

$x=1 \Rightarrow t=2$

$x=6 \Rightarrow t=3$

$$\Rightarrow 2\int_2^3 2\,dt = 4t\Big|_2^3 = 4(3-2) = 4.$$

c) $\boxed{a=?,\ a\in\mathbb{R}\ \text{a.î}\ \int_{-2}^{1} f'(x)\left(1+\frac{1}{f(x)}\right)dx = \ln(2e^a)}$

$$\int_{-2}^{1}\left[f'(x) + \frac{f'(x)}{f(x)}\right]dx = \int_{-2}^{1} f'(x)\,dx + \int_{-2}^{1}\frac{f'(x)}{f(x)}\,dx =$$

$$= f(x)\Big|_{-2}^{1} + \ln|f(x)|\Big|_{-2}^{1} = \left[f(1) - f(-2)\right] +$$

$$+ \left(\ln|f(1)| - \ln|f(-2)|\right)$$

$$f(1) = 1+2+\frac{2}{\sqrt{1+3}} = 3+\frac{2}{2} = 4$$

$$f(-2) = -2+2+\frac{2}{\sqrt{-2+3}} = 2$$

$$I = (4-2) + (\ln 4 - \ln 2) = 2 + \ln 2$$

$$\ln(2e^a) = 2+\ln 2 \Rightarrow \cancel{\ln 2} + \ln e^a = 2 + \cancel{\ln 2}$$

$$\Rightarrow \boxed{a=2}$$