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SIMULARE BAC 2026

ST- NAT

1. Se consideră funcția $f:(0,+\infty)\to\mathbb{R}$, $f(x)=\dfrac{1}{x}-1+4\ln(x+3)$.

a) Arătați că $f'(x)=\dfrac{(x-1)(4x+3)}{x^2(x+3)}$, $x\in(0,+\infty)$.

b) Arătați că $\lim\limits_{x\to1}\dfrac{4\ln(x+3)-f(x)}{\ln x}=1$.

a) $$f'(x)=\left[\frac{1}{x}-1+4\ln(x+3)\right]'=\frac{\cancel{1'\cdot x}-1\cdot x'}{x^2}-0+4\cdot\frac{(x+3)'}{x+3}=$$

$$=-\frac{1}{x^2}+\frac{4}{x+3}=\frac{-x-3+4x^2}{x^2(x+3)}=$$

$$=\frac{4x^2-4x+3x-3}{x^2(x+3)}=\frac{4x(x-1)+3(x-1)}{x^2(x+3)}=$$

$$=\frac{(x-1)(4x+3)}{x^2(x+3)}$$

b) $$\lim_{x\to1}\frac{\cancel{4\ln(x+3)}-\frac{1}{x}+1-\cancel{4\ln(x+3)}}{\ln x}=$$

$$=\lim_{x\to1}\frac{1-\frac{1}{x}}{\ln x}\ \left[\frac{0}{0}\right]\ \lim_{x\to1}\frac{\left(1-\frac{1}{x}\right)'}{(\ln x)'}=\lim_{x\to1}\frac{\frac{1}{x^2}}{\frac{1}{x}}=$$

$$= \lim_{x \to 1} \frac{1}{x} = 1$$

c) Demonstrați că $\dfrac{1}{x^2} - \dfrac{1}{x} \geq 4\ln\dfrac{x+3}{x^2+3}$, pentru orice $x \in (0,1]$.

$$f'(x) = 0 \Rightarrow 4x^2 - x - 3 = 0,\ \Delta = 49 \left\langle \begin{array}{l} x_1 = -\frac{3}{4} \\ x_2 = 1. \end{array} \right.$$

x	0		1		$+\infty$
$f'(x)$	$-\ -\ -\ -$		0	$+\ +$	$+$
$f(x)$		$\searrow$		$\nearrow$	

$x \in (0,1]$ $\qquad\qquad$ $\frac{1}{2};\ \frac{1}{4} \in (0,1]$

$$\frac{1}{2} \geq \frac{1}{4}$$

Generalizând, obținem: $\quad x \geq x^2$

$$\left.\begin{array}{l} x \geq x^2 \\ f\ s.d \\ x \in (0,1] \end{array}\right| \Rightarrow f(x) \leq f(x^2)$$

$$\frac{1}{x} - \cancel{1} + 4\ln(x+3) \leq \frac{1}{x^2} - \cancel{1} + 4\ln(x^2+3)$$

$$\frac{1}{x^2} - \frac{1}{x} \geq 4\ln(x+3) - 4\ln(x^2+3)$$

$$\boxed{\ln a - \ln b = \ln\frac{a}{b}}$$

$$\frac{1}{x^2} - \frac{1}{x} \geq 4\ln\frac{x+3}{x^2+3}$$