S1

Să se determine mulțimea valorilor parametrului real m pentru care matricea $\begin{pmatrix} 2 & x & 3 \\ m & x-1 & 1 \\ 1 & 1 & x \end{pmatrix}$ are rangul trei pentru orice $x < 0$. **(9 pct.)**

a) $(\frac{2}{3}, 1)$; b) $[\frac{5}{3}, 2]$; c) $(\frac{1}{2}, 2)$; d) $[-1, \frac{4}{3}] \cup \{2\}$; e) $[-\frac{1}{3}, 2]$; f) $(\frac{1}{3}, 1)$.

Fie $A = \begin{pmatrix} 2 & x & 3 \\ m & x-1 & 1 \\ 1 & 1 & x \end{pmatrix}$; $n=3 \Leftrightarrow \det A \neq 0$

$$\det A = \begin{vmatrix} 2 & x & 3 \\ m & x-1 & 1 \\ 1 & 1 & x \end{vmatrix} = 2(x^2-x) + 3m + x - 3(x-1) - 2 - mx^2 =$$

$$= 2x^2 - 2x + 3m + x - 3x + 3 - 2 - mx^2 =$$

$$= (2-m)x^2 - 4x + (3m+1)$$

$(2-m)x^2 - 4x + 3m + 1 = 0 \Rightarrow 2x^2 - mx^2 - 4x + 3m + 1 = 0 \Rightarrow$

$\Rightarrow m(3-x^2) = -2x^2 + 4x - 1 \Rightarrow m = \underbrace{\frac{-2x^2+4x-1}{3-x^2}}_{f(x)}$

C.E.: $3 - x^2 \neq 0 \Rightarrow x^2 \neq 3 \Rightarrow x \in \mathbb{R} \setminus \{\pm\sqrt{3}\}$

$f: \mathbb{R} \setminus \{-\sqrt{3}, \sqrt{3}\} \to \mathbb{R}$, $f(x) = \frac{-2x^2+4x-1}{3-x^2}$, $f(x) = m$

$$f'(x) = \frac{(-4x+4)(3-x^2) - (-2x^2+4x-1)\cdot(-2x)}{(3-x^2)^2} =$$

$$= \frac{-12x + 12 + \cancel{4x^3} - 4x^2 - \cancel{4x^3} + 8x^2 - 2x}{(3-x^2)^2} = \frac{4x^2 - 14x + 12}{(3-x^2)^2}$$

$f'(x) = 0 \Rightarrow 4x^2 - 14x + 12 = 0 \mid :2 \Rightarrow 2x^2 - 7x + 6 = 0$

$\Delta = 1$, $x_1 = \frac{7-1}{4} = \frac{3}{2}$, $x_2 = 2$

x	$-\infty$		$-\sqrt{3}$		0	$\frac{3}{2}$	$\sqrt{3}$	2	$+\infty$
$f'(x)$		$+$	$+$	$+$	$+\ 0$	$-$	0	$+$	$+$
$f(x)$	2	$\nearrow$	$+\infty$ $\mid$ $-\infty$	$\nearrow$	$-\frac{1}{3}$				

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{-2x^2+4x-1}{3-x^2} = 2$$

$$l_s(-\sqrt{3}) = \lim_{\substack{x\to-\sqrt{3}\\ x<-\sqrt{3}}} \frac{-2x^2+4x-1}{3-x^2} = \frac{-6-4\sqrt{3}-1}{0_-} = +\infty$$

$$l_d(-\sqrt{3}) = -\infty$$

$$l_1(0) = -\frac{1}{3}$$

$$\Rightarrow \operatorname{Im} f = \left(-\infty; -\frac{1}{3}\right) \cup (2; +\infty)$$

$$f(x) \neq m \Rightarrow m \in \left[-\frac{1}{3}; 2\right]$$