Model test-Asimptote

(1) Fie $f: D \to \mathbb{R}$, $f(x) = \dfrac{x^2-x-4}{x^2-9}$

a) $D = ?$

C.E: $x^2-9 \neq 0 \Rightarrow x^2 \neq 9 \Rightarrow x \neq \pm 3 \Rightarrow$

$\Rightarrow x \in \mathbb{R} \setminus \{-3, 3\} \Rightarrow D = \mathbb{R} \setminus \{-3, 3\}$

b) Determinați asimptotele la graficul fct f.

Căutăm A.V (asimptote verticale)

$x = -3$ $\quad l_s(-3) = \lim\limits_{\substack{x \to -3 \\ x < -3}} f(x) = \lim\limits_{\substack{x \to -3 \\ x < -3}} \dfrac{x^2-x-4}{x^2-9} = \dfrac{9+3-4}{9-9} =$

$\Rightarrow l_s(-3) = \dfrac{8}{0_?}$

x	$-\infty$		-3		3		$+\infty$
x^2-9		$+$	0	$-$	0	$+$	

$l_s(-3)$ $\quad$ $l_d(-3)$

$l_s(-3) = \dfrac{8}{0_+} = +\infty$

$l_d(-3) = \lim\limits_{\substack{x \to -3 \\ x > -3}} f(x) = \lim\limits_{\substack{x \to -3 \\ x > -3}} \dfrac{x^2-x-4}{x^2-9} = \dfrac{8}{0_-} = -\infty$

Cum $\begin{matrix} l_s(-3) = +\infty \\ l_d(-3) = -\infty \end{matrix} \Rightarrow x = -3$ A.V.

$\boxed{x=3} \Rightarrow l_s(3) = \lim\limits_{\substack{x\to 3\\ x<3}} \frac{x^2-x-4}{x^2-9} = \frac{9-3-4}{0_-} = \frac{2}{0_-} = -\infty$

$l_d(3) = \frac{2}{0_+} = +\infty \Rightarrow$

$\Rightarrow \boxed{x=3}$ A.V.

Căutăm A.O$_z$ (asimptote orizontale)

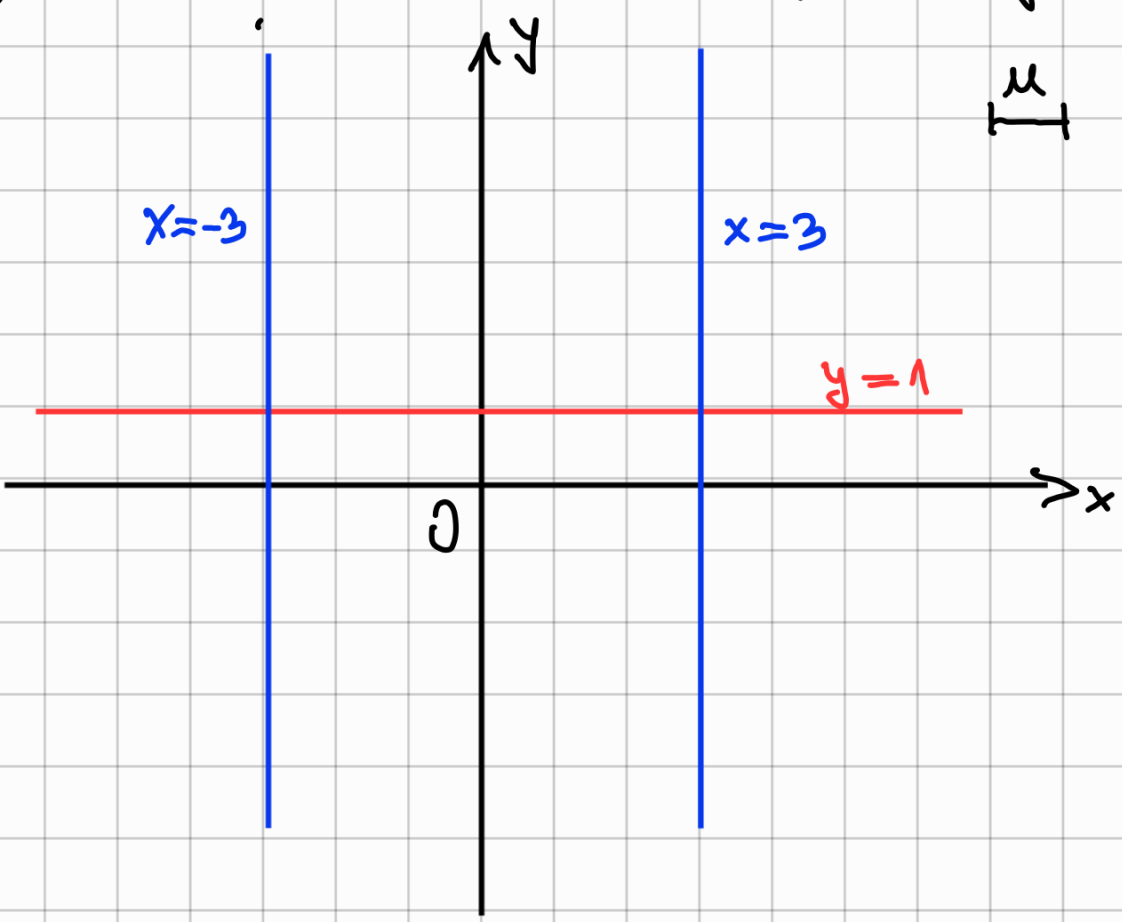
$y = \lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \frac{x^2-x-4}{x^2-9} = \frac{1}{1} = 1$ (ac grad)

$\Rightarrow \boxed{y=1}$ A.O$_z$ la $+\infty$

$y = \lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \frac{x^2-x-4}{x^2-9} = \frac{1}{1} = 1$ (ac grad)

$\Rightarrow \boxed{y=1}$ A.O$_z$ la $-\infty$.

c) Reprezentați geometric asimptotele găsite.



② Fie $f: \mathbb{R} \to \mathbb{R}$, $f(x) = \frac{x^3}{x^2+2}$

a) Determinați asimptota la ∞ la gf.

$y = \lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^3}{x^2+2} = -\infty \Rightarrow$

$x^3 \to$ grad 3, $x^2+2 \to$ grad 2

$\Rightarrow$ nu avem A.O.

$y = mx+n$

$m = \lim_{x\to\infty} \frac{f(x)}{x}$

$n = \lim_{x\to\infty} [f(x) - mx]$

$$m = \lim_{x\to\infty} \frac{\frac{x^3}{x^2+2}}{\frac{x}{1}} = \lim_{x\to\infty} \frac{x^3}{x^2+2} \cdot \frac{1}{x} = \lim_{x\to\infty} \frac{x^2}{x^2+2} = \frac{1}{1} = 1$$

(ac grad)

$\boxed{m=1}$

$$n = \lim_{x\to\infty} (f(x) - 1\cdot x) = \lim_{x\to\infty} \left(\frac{x^3}{x^2+2} - x\right) = \lim_{x\to\infty} \frac{x^3 - x^3 - 2x}{x^2+2} = \lim_{x\to\infty} \frac{-2x}{x^2+2} = 0$$

$-2x \to$ grad 1, $x^2+2 \to$ grad 2

$\boxed{n=0}$

$\Rightarrow y = 1\cdot x + 0 \Rightarrow \boxed{y = x}$ A.Obl la $+\infty$

b) Reprezentați geometric asimptota găsită

$y = x$

Luam $x = 0 \Rightarrow y = 0 \Rightarrow A(0,0)$

Luam $y = 1 \Rightarrow x = 1 \Rightarrow B(1,1)$