Funcții trigonometrice directe

$\sin(a+b) = \sin a \cos b + \sin b \cos a$

$\sin(a-b) = \sin a \cos b - \sin b \cos a$ (+)

$$\sin(\underbrace{a+b}_{\text{not } x}) + \sin(\underbrace{a-b}_{\text{not } y}) = 2\sin a \cos b \quad (1)$$

$$\begin{cases} a+b=x \\ a-b=y \end{cases} \xrightarrow{(+)} 2a = x+y \Rightarrow \boxed{a = \frac{x+y}{2}}$$

$$\frac{x+y}{2} + b = x \Rightarrow b = x - \frac{x+y}{2} \Rightarrow b = \frac{2x-x-y}{2}$$

$$\Rightarrow \boxed{b = \frac{x-y}{2}}$$

Înlocuind în rel. (1), obținem:

$$\boxed{\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}}$$

$$\sin x - \sin y = 2\sin\frac{x-y}{2}\cos\frac{x+y}{2}$$

$\cos(a+b) = \cos a\cos b - \sin a\sin b$

$\cos(a-b) = \cos a\cos b + \sin a\sin b$ (+)

$\cos(a+b) + \cos(a-b) = 2\cos a\cos b$

$\cos(a+b) - \cos(a-b) = -2\sin a\sin b$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

Exerciții rezolvate:

(1) $\sin 75^\circ + \sin 15^\circ = ?$

$$\sin a + \sin b = 2\sin\frac{a+b}{2}\cos\frac{a-b}{2}$$

$$\sin 75^\circ + \sin 15^\circ = 2\sin\frac{75^\circ+15^\circ}{2}\cos\frac{75^\circ-15^\circ}{2} =$$

$$= 2\sin 45^\circ \cdot \cos 30^\circ = 2\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{6}}{2}$$

(2) $\sin\frac{\pi}{12} - \sin\frac{5\pi}{12}$

$$\sin a - \sin b = 2\sin\frac{a-b}{2}\cos\frac{a+b}{2}$$

$$\sin\frac{\pi}{12} - \sin\frac{5\pi}{12} = 2\sin\frac{\frac{\pi}{12} - \frac{5\pi}{12}}{2}\cos\frac{\frac{\pi}{12} + \frac{5\pi}{12}}{2} =$$

$$= 2\sin\left(\frac{-4\pi}{12}\cdot\frac{1}{2}\right)\cdot\cos\left(\frac{6\pi}{12}\cdot\frac{1}{2}\right) = 2\sin\left(-\frac{\pi}{6}\right)\cdot\cos\frac{\pi}{4}$$

$\sin(-x) = -\sin x$

$$= -2\sin\frac{\pi}{6}\cos\frac{\pi}{4}$$

$$= -\cancel{2}\cdot\frac{1}{\cancel{2}}\cdot\frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}$$

Exercitii propuse

(1) Să se transforme în produs:

a) $\cos 75^\circ + \cos 15^\circ$

b) $\cos 75^\circ - \cos 15^\circ$

c) $\sin\frac{\pi}{10} + \sin\frac{3\pi}{10}$

d) $\cos\frac{2\pi}{3} + \cos\frac{7\pi}{3}$

e) $\sin x - \sin\left(\frac{\pi}{4} - x\right)$

f) $\sin x + \cos x$

g) $\sin x - \cos x$

h) $\sin x + \sin 2x + \sin 3x$