



Aplicații - Combinații

1. Calculați:
- a) C_6^3
 - b) $C_5^2 - C_5^3$
 - c) $\frac{C_8^4}{C_7^4}$

Rezolvare:

$$a) C_6^3 = \frac{6!}{3!(6-3)!} = \frac{6!}{3! \cdot 3!} = \frac{\cancel{2!} \cdot 4 \cdot 5 \cdot \cancel{6^1}}{\cancel{3!} \cdot 1 \cdot 2 \cdot 3} = 4 \cdot 5 = 20.$$

$$b) C_5^2 - C_5^3 = C_5^{5-2} - C_5^3 = C_5^3 - C_5^3 = 0$$

$C_m^k = C_m^{m-k}$

 - formula combinatorilor complementare

$$c) \frac{C_8^4}{C_7^4} = \frac{\frac{8!}{4! \cdot (8-4)!}}{\frac{7!}{4! \cdot (7-4)!}} = \frac{\frac{8!}{\cancel{4!} \cdot 4!}}{\frac{7!}{\cancel{4!} \cdot 3!}} = \frac{8!}{7!} \cdot \frac{3!}{4!} = \frac{8}{4} = 2$$

Exerciții propuse

- Calculați:
- | | | |
|---------------|--------------------|-----------------------------------|
| a) C_7^5 | c) $C_2^1 - C_3^2$ | f) $\frac{C_{11}^2}{C_{10}^3}$ |
| b) C_9^2 | d) $C_7^2 - C_7^5$ | g) $\frac{C_{15}^{12}}{C_{15}^3}$ |
| c) C_{10}^4 | e) $C_3^1 + C_3^2$ | |

2. Calculate: a) $\frac{C_7^3 - C_7^1}{C_7^2}$

b) $\frac{C_9^5 - C_9^4}{C_8^4 - C_8^3}$

Resolve:

a) $\frac{C_7^3 - C_7^1}{C_7^2} = \frac{\frac{7!}{3!(7-3)!} - \frac{7!}{1!(7-1)!}}{\frac{7!}{2!(7-2)!}} = \frac{\frac{1}{3! \cdot 4!} - \frac{1}{6!}}{\frac{1}{2! \cdot 5!}} =$

$= \frac{\frac{1}{6 \cdot 4!} - \frac{1}{1! \cdot 5 \cdot 6}}{\frac{1}{2 \cdot 4! \cdot 5}} = \frac{\frac{1}{6} - \frac{1}{5 \cdot 6}}{\frac{1}{2 \cdot 5}} = \left(\frac{1}{6} - \frac{1}{30} \right) \cdot \frac{2 \cdot 5}{1} =$

$= \frac{4}{3} \cdot \frac{10}{1} = \frac{4}{3}$

b) $\frac{C_9^5 - C_9^3}{C_8^4 - C_8^3} = \frac{\frac{9!}{5!(9-5)!} - \frac{9!}{3!(9-3)!}}{\frac{8!}{4!(8-4)!} - \frac{8!}{3!(8-3)!}} =$

$= \frac{\frac{9!}{5! \cdot 4!} - \frac{9!}{3! \cdot 6!}}{\frac{8!}{4! \cdot 4!} - \frac{8!}{3! \cdot 5!}} = \frac{\frac{8! \cdot 9}{3! \cdot 4 \cdot 5 \cdot 4!} - \frac{8! \cdot 9}{3! \cdot 4! \cdot 5 \cdot 6}}{\frac{8!}{3! \cdot 4 \cdot 4!} - \frac{8!}{3! \cdot 4! \cdot 5}} = \frac{\frac{9}{4 \cdot 5} - \frac{9}{5 \cdot 6}}{\frac{1}{4} - \frac{1}{5}}$

$= \frac{\frac{9}{5} \left(\frac{1}{4} - \frac{1}{6} \right)}{\frac{1}{4} - \frac{1}{5}} = \frac{9}{5} \cdot \frac{\frac{1}{12}}{\frac{1}{20}} = \frac{9}{5} \cdot \frac{1}{12} \cdot \frac{20}{1} = \frac{9}{3} = 3$

Exercitiu propriu.

Calculati:

a) $\frac{C_6^4 - C_6^1}{C_6^3}$

c) $\frac{C_{10}^7 - C_{10}^6}{C_7^5 - C_7^3}$

b) $\frac{C_3^1 - C_2^1}{C_4^1}$

d) $\frac{C_{12}^6 - C_{12}^5}{C_6^5 - C_6^4}$

3. Calculati:

a) C_m^2

c) $C_m^2 - C_m^1$

b) $\frac{C_m^4}{C_m^3}$

d) $\frac{C_m^3 - C_m^1}{C_m^2}$

Rezolvare:

a) $C_m^2 = \frac{n!}{2!(n-2)!} = \frac{\cancel{(n-2)!} \cdot (n-1) \cdot n}{2 \cdot \cancel{(n-2)!}} = \frac{(n-1) \cdot n}{2}$

b) $\frac{C_m^4}{C_m^3} = \frac{\frac{n!}{4!(n-4)!}}{\frac{n!}{3!(n-3)!}} = \frac{1}{4!(n-4)!} \cdot \frac{3! \cdot (n-3)!}{1} =$

$= \frac{3!}{4!} \cdot \frac{(n-3)!}{(n-4)!} = \frac{\cancel{3!} \cdot 1}{\cancel{3!} \cdot 4} \cdot \frac{\cancel{(n-4)!} \cdot (n-3)}{\cancel{(n-4)!}} = \frac{n-3}{4}$

c) $C_m^2 - C_m^1 = \frac{n!}{2!(n-2)!} - \frac{n!}{1!(n-1)!} = \frac{n!}{2(n-2)!} - \frac{n!}{(n-1)!} =$

$= \frac{\cancel{(n-2)!} \cdot (n-1) \cdot n}{2 \cdot \cancel{(n-2)!}} - \frac{\cancel{(n-1)!} \cdot n}{\cancel{(n-1)!}} = \frac{(n-1)n}{2} - n = n \left(\frac{n-1}{2} - 1 \right)$

$= n \cdot \frac{n-1-2}{2} = \frac{n(n-3)}{2}$

$$\begin{aligned}
 d) \quad \frac{C_m^3 - C_m^1}{C_m^2} &= \frac{\frac{m!}{3!(m-3)!} - \frac{m!}{1!(m-1)!}}{\frac{m!}{2!(m-2)!}} = \frac{\frac{1}{6 \cdot (m-3)!} - \frac{1}{(m-3)!(m-2)(m-1)}}{\frac{1}{2(m-3)!(m-2)}} \\
 &= \frac{\frac{(m-2)(m-1)}{6} - \frac{1}{1}}{\frac{1}{2(m-2)}} = \frac{(m-2)(m-1) - 6}{\frac{1}{2(m-2)}} \cdot \frac{2(m-2)}{1} = \\
 &= \frac{m^2 - 2m - m + 2 - 6}{3} = \frac{m^2 - 3m - 4}{3}
 \end{aligned}$$

Exercitiu propus

Calculat:

a) C_m^1

b) C_m^3

c) $C_m^1 - C_m^0$

d) $\frac{C_m^7}{C_m^5}$

e) $\frac{C_m^4 - C_m^2}{C_m^3}$

f) C_{m+1}^2

g) C_{m+2}^1

h) C_{m+3}^2