



Să se rezolve în $M_2(\mathbb{R})$, ecuația $A \cdot X = B$

$$A = \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}, B = \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} \text{ și } X \in M_2(\mathbb{R}).$$

Rezolvare:

met1 $A^{-1} \cdot A \cdot X = B \Rightarrow A^{-1} \cdot A X = A^{-1} \cdot B$

$$I_2 \cdot X = A^{-1} \cdot B$$

$$X = A^{-1} \cdot B.$$

$$\det A = \begin{vmatrix} 2 & -1 \\ -3 & 2 \end{vmatrix} = 2 \cdot 2 - (-3) \cdot (-1) = 4 - 3 = 1 \neq 0 \Rightarrow$$
$$\Rightarrow A - \text{invertibilă} \Rightarrow \exists A^{-1} \in M_2(\mathbb{R})$$

$$A^{-1}(x) = \frac{1}{\det A} \cdot A^*$$

$$A^t = \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}, A^* = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$\left. \begin{aligned} a_{11} &= (-1)^{1+1} \cdot 2 = 2 \\ a_{12} &= (-1)^{1+2} \cdot (-1) = 1 \\ a_{21} &= (-1)^{2+1} \cdot (-3) = 3 \\ a_{22} &= (-1)^{2+2} \cdot 2 = 2 \end{aligned} \right\} \Rightarrow A^* = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \Rightarrow A^{-1} = \frac{1}{1} \cdot \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \Rightarrow$$
$$\Rightarrow A^{-1} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$$

$$X = A^{-1} \cdot B = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 2+0 & 6-2 \\ 3+0 & 9-4 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}$$

met 2 : Fie $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_2(\mathbb{R})$

$$A \cdot X = B \Rightarrow \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} \Rightarrow$$
$$\Rightarrow \begin{pmatrix} 2a-c & 2b-d \\ -3a+2c & -3b+2d \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} \Rightarrow \begin{cases} 2a-c=1 \\ -3a+2c=0 \end{cases} \textcircled{1}$$
$$\begin{cases} 2b-d=3 \\ -3b+2d=-2 \end{cases} \textcircled{2}$$

①

$$\begin{cases} 2a-c=1 \\ -3a+2c=0 \end{cases} \quad | \cdot 2 \Rightarrow \begin{cases} 4a-2c=2 \\ -3a+2c=0 \end{cases} \quad \oplus$$
$$a = 2 \Rightarrow a=2$$
$$2 \cdot 2 - c = 1 \Rightarrow 4 - c = 1 \Rightarrow -c = 1 - 4 \Rightarrow c=3$$

②

$$\begin{cases} 2b-d=3 \\ -3b+2d=-2 \end{cases} \quad | \cdot 2 \Rightarrow \begin{cases} 4b-2d=6 \\ -3b+2d=-2 \end{cases} \quad \oplus$$
$$b = 4 \Rightarrow b=4$$
$$2 \cdot 4 - d = 3 \Rightarrow -d = 3 - 8 \Rightarrow d=5$$

$$\Rightarrow X = \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}$$

Pentru mai multe probleme rezolvate accesați:

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