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Subiectul III. 1. – Tehnologic

1. Se consideră funcția $f:\mathbb{R}\to\mathbb{R}$, $f(x)=(x^2+x+1)e^x$

a) Arătați că $f'(x)=e^x(x+1)(x+2)$, $\quad x\in\mathbb{R}$

b) Calculați $\lim\limits_{x\to -1}\dfrac{f(x)-f(-1)}{x+1}$

c) Demonstrați că $x^2+x+1\le\dfrac{3}{e^{x+2}}$, pentru orice $x\in(-\infty,-1)$

$$(f\cdot g)'=f'\cdot g+f\cdot g'$$

$$\lim_{x\to a}\frac{f(x)-f(a)}{x-a}=f'(a)$$

a) $f'(x)=(x^2+x+1)'\cdot e^x+(x^2+x+1)\cdot(e^x)'=$

$=(2x+1)\cdot e^x+(x^2+x+1)\cdot e^x=e^x(2x+1+x^2+x+1)=$

$=e^x(x^2+\underbrace{3x}_{2x+x}+2)=e^x(x^2+2x+x+2)=$

$=e^x[x(x+2)+1\cdot(x+2)]=e^x\cdot(x+2)(x+1).$

b) $\lim\limits_{x\to -1}\dfrac{f(x)-f(-1)}{x+1}=f'(-1)=e^{-1}(-1+2)\cdot\underbrace{(-1+1)}_{0}=0.$

c) $f'(x)=0\Rightarrow \underbrace{e^x}_{>0}(x+1)(x+2)=0\Rightarrow x+1=0$ sau $x+2=0$

$\Rightarrow x=-1$ sau $x=-2$

x	$-\infty$		-2		-1		$+\infty$
$f'(x)$		$+$	0	$-\,-\,-$	0	$+$	
$f(x)$		$\nearrow$	$\frac{3}{e^2}$	$\searrow$		$\nearrow$	

$f(-2)=[(-2)^2+(-2)+1]\cdot e^{-2}=$

$=3e^{-2}=\dfrac{3}{e^2}$

$\nearrow\ 2\ \searrow\ \Rightarrow A\left(-2,\dfrac{3}{e^2}\right)$ este punct de maxim local

$$\Rightarrow \quad f(x) \leq f(-2)$$

$$(x^2+x+1) \cdot e^x \leq \frac{3}{e^2} \quad \Big| : e^x$$

$$\frac{(x^2+x+1) \cdot \cancel{e^x}}{\cancel{e^x}} \leq \frac{3}{e^2} \cdot \frac{1}{e^x}$$

$$\boxed{a^x \cdot a^y = a^{x+y}}$$

$$x^2+x+1 \leq \frac{3}{e^{x+2}}$$