Puncte critice - Aplicații

① Fie funcția $f: D \to \mathbb{R}$, $f(x) = x^2 - 4x + 3$

a) Determinați domeniul de definiție al fct f (D)

b) Determinați domeniul de derivabilitate al funcției f. ($D_{f'}$)

c) Rezolvați ecuația $f'(x) = 0$.

Rezolvare:

a) $f(x) = x^2 - 4x + 3$ - fct polinomială $\Rightarrow D = \mathbb{R}$

b) $f'(x) = (x^2 - 4x + 3)' = 2x - 4$ - fct polinomială

$\Rightarrow D_{f'} = \mathbb{R}$

c) $\left.\begin{array}{l} f'(x) = 0 \\ f'(x) = 2x - 4 \end{array}\right| \Rightarrow 2x - 4 = 0$

$2x = 4$

$x = 2$ $\Rightarrow$ f are punct critic în $x = 2$

$x = 2 \Rightarrow y = 2^2 - 4 \cdot 2 + 3$

$y = 4 - 8 + 3$

$y = -1$ $\Rightarrow P(2, -1)$ - punct critic

② Fie $f: D \to \mathbb{R}$, $f(x) = \dfrac{x^2}{x-2}$

a) $D = ?$ b) $D_{f'} = ?$ c) $f'(x) = 0$

Rezolvare

a) $f(x) = \dfrac{x^2}{x-2}$

C.E: $x - 2 \neq 0 \Rightarrow x \neq 2 \Rightarrow D = \mathbb{R} \setminus \{2\}$

b) $f'(x) = \left(\frac{x^2}{x-2}\right)' = \frac{(x^2)' \cdot (x-2) - x^2 \cdot (x-2)'}{(x-2)^2} = \frac{2x(x-2) - x^2}{(x-2)^2} =$

$= \frac{2x^2 - 4x - x^2}{(x-2)^2} = \frac{x^2-4x}{(x-2)^2}$

CE: $x-2 \neq 0 \Rightarrow x \neq 2 \Rightarrow D_{f'} = \mathbb{R} \setminus \{2\}$

c) $f'(x) = 0 \Rightarrow \frac{x^2-4x}{\underbrace{(x-2)^2}_{>0}} = 0 \Rightarrow x^2-4x=0$

$x(x-4)=0$ ⟨ $x=0$, $x=4$

$x=0 \Rightarrow y = \frac{0^2}{0-2} = 0 \Rightarrow A(0,0)$ pct critic

$x=4 \Rightarrow y = \frac{4^2}{4-2} = \frac{16}{2} = 8 \Rightarrow B(4,8)$ pct critic

3 $f: D \to \mathbb{R}$, $f(x) = (x^2+6x-15)e^x$

a) $D = ?$ b) $D_{f'} = ?$ c) $f'(x) = 0$.

Rezolvare:

a) $f(x) = (x^2+6x-15)e^x \Rightarrow D = \mathbb{R}$

b) $f'(x) = (x^2+6x-15)' \cdot e^x + (x^2+6x-15) \cdot (e^x)' =$

$= (2x+6) \cdot e^x + (x^2+6x-15) \cdot e^x =$

$= e^x(2x+6+x^2+6x-15) = e^x(x^2+8x-9) \Rightarrow D_{f'} = \mathbb{R}$

c) $f'(x) = 0 \Rightarrow \underbrace{e^x}_{>0}(x^2+8x-9) = 0 \Rightarrow x^2+8x-9=0$

$\Delta = 64+36 = 100$

$x_1 = \frac{-8-10}{2} = -9$

$x_2 = \frac{-8+10}{2} = 1$

$\Rightarrow A(-9, f(-9))$ și $B(1, f(1))$ puncte critice.

4 Fie $f: D \to \mathbb{R}$, $f(x) = x \ln x$

a) $D = ?$ b) $D_{f'} = ?$ c) $f'(x) = 0$.

Rezolvare:

a) $f(x) = x \ln x$

C.E: $x > 0 \Rightarrow \boxed{D = (0, +\infty)}$

b) $f'(x) = (x \ln x)' = x' \ln x + x \cdot (\ln x)' = 1 \cdot \ln x + \cancel{x} \cdot \frac{1}{\cancel{x}} =$

$= \ln x + 1$

C.E: $x > 0 \Rightarrow \boxed{D_{f'} = (0, +\infty)}$

c) $f'(x) = 0 \Rightarrow \ln x + 1 = 0 \Rightarrow \ln x = -1 \Rightarrow x = e^{-1} \Rightarrow x = \frac{1}{e}$

$x = \frac{1}{e} \Rightarrow y = \frac{1}{e} \ln \frac{1}{e} = \frac{1}{e} \ln e^{-1} = -\frac{1}{e} \underbrace{\ln e}_{1} = -\frac{1}{e}$

$\boxed{P\left(\frac{1}{e}, -\frac{1}{e}\right)}$ pct critic