Rezolvare simulare BAC - Tehnologic 19 mai 2026

Subiectul I

1) $2\cdot(2-\sqrt{3})(2+\sqrt{3})\cdot\left(\frac{1}{2}\right)^{-1} = 2\cdot(2^2-\sqrt{3}^2)\cdot 2 = 2\cdot(4-3)\cdot 2 =$

$(a-b)(a+b)=a^2-b^2$

$= 2\cdot 1\cdot 2 = 4.$

$\left(\frac{1}{a}\right)^{-1}=a$

2) $f:\mathbb{R}\to\mathbb{R},\ f(x)=2x-1.$ $m=?$ a.î. $A(1,2-m)\in G_f$

$A(1,2-m)\in G_f \Rightarrow f(1)=2-m$

$f(1)=2\cdot 1-1=1$

$\Rightarrow 2-m=1$

$-m=1-2$

$-m=-1 \quad |\cdot(-1)$

$\boxed{m=1}$

3) $\log_3(2x-5)=2$

C.E: $2x-5>0 \Rightarrow 2x>5 \Rightarrow x>\frac{5}{2}$

$\log_a x=b \Rightarrow x=a^b$

$\log_3(2x-5)=2 \Rightarrow 2x-5=3^2 \Rightarrow 2x=9+5 \Rightarrow 2x=14 \Rightarrow$

$\Rightarrow \boxed{x=7}$

Verificare: $2\cdot 7-5=14-5=9>0$ A

sau $7>\frac{5}{2}$ (A)

4) P.i $\stackrel{\text{not}}{=} x$ $\qquad$ P.f. = 720 lei

scumpirea $= \frac{20}{100}\cdot x = \frac{1}{5}x = \frac{x}{5}$

$P.f = P.i + S \Rightarrow 720 = x + \frac{x}{5} \quad / \cdot 5 \Rightarrow$

$\Rightarrow 3600 = 5x + x \Rightarrow 6x = 3600 \Rightarrow x = 600$ lei

5. $A(1,1)$, $B(5,4)$, $C(4,-3)$

$P_{\Delta ABC} = ?$

$P_{\Delta ABC} = AB + BC + CA$

$AB = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} = \sqrt{(-4)^2 + (-3)^2} = 5$

$BC = \sqrt{1^2 + 7^2} = \sqrt{50} = 5\sqrt{2}$

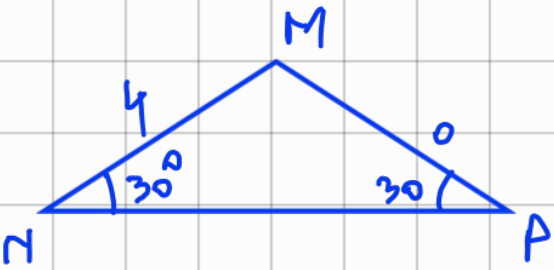
$AC = \sqrt{(-3)^2 + 4^2} = 5$

$P_{\Delta ABC} = 5 + 5\sqrt{2} + 5 = 10 + 5\sqrt{2} = 5(2 + \sqrt{2})$

6. ΔMNP, $MN = 4$

$m\hat{N} = m\hat{P} = 30^\circ$

$A_{\Delta MNP} = ?$



$\hat{N} = \hat{P} \Rightarrow \Delta MNP$ isoscel, $MN = 4$ $\Rightarrow MP = 4.$

$\sin 120^\circ = \sin(180^\circ - 60^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$

$\hat{M} + \hat{N} + \hat{P} = 180^\circ \Rightarrow m\hat{M} = 120^\circ$

$A_{\Delta MNP} = \frac{MN \cdot MP \cdot \sin M}{2} = \frac{4 \cdot \cancel{4}^{\,2} \cdot \sin 120^\circ}{\cancel{2}} = 8 \cdot \frac{\sqrt{3}}{2} = 4\sqrt{3}$

Subiectul II

1. $A(x) = \begin{pmatrix} 1 & x-1 \\ x+2 & -1 \end{pmatrix}$, $x \in \mathbb{R}$

a) $A(-2) = \begin{pmatrix} 1 & -3 \\ 0 & -1 \end{pmatrix} \Rightarrow \det A(-2) = \begin{vmatrix} 1 & -3 \\ 0 & -1 \end{vmatrix} = -1 + 0 = -1$

b) $A(3) \cdot A(1) - 5A(0) = ?$

$$A(3)\cdot A(1)=\begin{pmatrix}1 & 2\\ 5 & -1\end{pmatrix}\cdot\begin{pmatrix}1 & 0\\ 3 & -1\end{pmatrix}=\begin{pmatrix}7 & -2\\ 2 & 1\end{pmatrix}$$

$$5A(0)=5\cdot\begin{pmatrix}1 & -1\\ 2 & -1\end{pmatrix}=\begin{pmatrix}5 & -5\\ 10 & -5\end{pmatrix}$$

$$A(3)\cdot A(1)-5A(0)=\begin{pmatrix}7 & -2\\ 2 & 1\end{pmatrix}-\begin{pmatrix}5 & -5\\ 10 & -5\end{pmatrix}=\begin{pmatrix}2 & 3\\ -8 & 6\end{pmatrix}$$

c) $x=?$, $x\in\mathbb{R}$ a.î. $\det\left(A(x)+2I_2\right)\geq 3$

$$A(x)+2I_2=\begin{pmatrix}1 & x-1\\ x+2 & -1\end{pmatrix}+\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}=\begin{pmatrix}3 & x-1\\ x+2 & 1\end{pmatrix}$$

$$\det\left(A(x)+2I_2\right)=\begin{vmatrix}3 & x-1\\ x+2 & 1\end{vmatrix}=3-(x-1)(x+2)=$$
$$=3-(x^2+2x-x-2)=-x^2-x+5$$

$-x^2-x+5\geq 3 \Rightarrow -x^2-x+2\geq 0$

$-x^2-x+2=0$

$\Delta=9$, $x_1=\frac{1-3}{-2}=1$, $x_2=\frac{1+3}{-2}=-2$

x	$-\infty$		-2		1		$+\infty$
$-x^2-x+2$		$-$	0	$+$	0	$-$	

$x\in[-2,1]$

2) $f=x^3+mx^2+2x-5$, $m\in\mathbb{R}$

a) $f(0)=0^3+m\cdot 0^2+2\cdot 0-5=-5$

b) $m=?$ a.î. 1 - răd a lui f.

$x_0=1$ răd $\Rightarrow f(1)=0$

$f(1)=1+m+2-5=m-2$

$\Rightarrow m-2=0 \Rightarrow m=2$

c) $m=?$, $m\in\mathbb{N}$ a.î. $x_1^2+x_2^2+x_3^2=5$

Se aplică rel. lui Viete.

$$\begin{cases} x_1+x_2+x_3 = -\frac{b}{a} = -m \\ x_1x_2+x_1x_3+x_2x_3 = \frac{c}{a} = 2 \\ x_1x_2x_3 = -\frac{d}{a} = 5 \end{cases}$$

$$x_1^2+x_2^2+x_3^2 = (x_1+x_2+x_3)^2 - 2(x_1x_2+x_1x_3+x_2x_3)$$

$$5 = (-m)^2 - 2\cdot 2$$

$$\left.\begin{array}{l} 5 = m^2-4 \Rightarrow m^2 = 9 \Rightarrow m = \pm 3 \\ m \in \mathbb{N} \end{array}\right| \Rightarrow \boxed{m=3}$$

Subiectul III

1) $f:\mathbb{R}\setminus\{1\}\to\mathbb{R}, \quad f(x) = \frac{x^2}{x-1}$

a) $f'(x) = \frac{(x^2)'(x-1) - x^2\cdot(x-1)'}{(x-1)^2} = \frac{2x(x-1)-x^2}{(x-1)^2} =$

$= \frac{2x^2-2x-x^2}{(x-1)^2} = \frac{x^2-2x}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2}$

b) $\lim\limits_{x\to\infty} \frac{f(x)}{1-2x} = \lim\limits_{x\to\infty} \frac{\frac{x^2}{x-1}}{1-2x} = \lim\limits_{x\to\infty} \frac{x^2}{(x-1)(1-2x)} =$

$= \lim\limits_{x\to\infty} \frac{x^2}{-2x^2+3x-1} \overset{\left[\frac{\infty}{\infty}\right]}{\underset{L'H}{=}} \lim\limits_{x\to\infty} \frac{2x}{-4x+3} \overset{\left[\frac{\infty}{\infty}\right]}{\underset{L'H}{=}} \frac{2}{-4} = -\frac{1}{2}$

c) $f(x) \geq 4, \ \forall x \in (1,+\infty)$

$f'(x) = 0 \Rightarrow \frac{x(x-2)}{(x-1)^2} = 0 \Rightarrow x(x-2) = 0 \Rightarrow x=0$ sau $x=2$

x	$-\infty$		0		1		2		$+\infty$
$f'(x)$		$+$	0	$-$	|	$-$	0	$+$	
$f(x)$		$\nearrow$		$\searrow$	|	$\searrow$		$\nearrow$	

$x \in (1, +\infty) \Rightarrow x_0 = 2$ pct de min. local $\Rightarrow$

$$\left.\begin{array}{l} \Rightarrow f(x) \geq f(2) \\ f(2) = \dfrac{2^2}{2-1} = 4 \end{array}\right| \Rightarrow f(x) \geq 4.$$

(2) $f:(0,+\infty) \to \mathbb{R}, \quad f(x) = \dfrac{4x^2-1}{x}$

a) $$\int_1^2 x^2 f(x)\,dx = \int_1^2 x^{\cancel{2}} \cdot \frac{4x^2-1}{\cancel{x}}\,dx = \int_1^2 x(4x^2-1)\,dx =$$

$$= \int_1^2 (4x^3 - x)\,dx = 4\int_1^2 x^3\,dx - \int_1^2 x\,dx = \cancel{4}\cdot\frac{x^4}{\cancel{4}}\Big|_1^2 - \frac{x^2}{2}\Big|_1^2 =$$

$$= (2^4 - 1^4) - \left(\frac{2^2}{2} - \frac{1^2}{2}\right) = 15 - \left(2 - \frac{1}{2}\right) = 15 - \frac{3}{2} = \frac{27}{2}$$

b) Fie $F:(0,+\infty) \to \mathbb{R}$ o primitivă a lui f a.î. $F(1) = 5$

$$F(x) = \int \frac{4x^2-1}{x}\,dx = \int\left(\frac{4x^{\cancel{2}}}{\cancel{x}} - \frac{1}{x}\right)dx = 4\int x\,dx - \int \frac{1}{x}\,dx =$$

$$= 4\frac{x^2}{2} - \ln\underbrace{|x|}_{>0} + C = 2x^2 - \ln x + C$$

$$F(1) = 5 \Rightarrow 2 - \underbrace{\ln 1}_{0} + C = 5$$

$$2 + C = 5$$

$$C = 3$$

$F:(0,+\infty) \to \mathbb{R}, \quad F(x) = 2x^2 - \ln x + 3$

c) F prim. a lui $f \Rightarrow F$ derivabilă pe $(0,+\infty)$

$$F'(x) = f(x), \quad \forall x \in (0,+\infty)$$

$$F''(x) = (F'(x))' = f'(x) = \left(\frac{4x^2-1}{x}\right)' = \frac{8x\cdot x - 4x^2 + 1}{x^2} =$$

$$= \frac{\overbrace{4x^2+1}^{>0}}{\underbrace{x^2}_{>0}} > 0 \Rightarrow F''(x) > 0 \Rightarrow F - \text{convexă}$$

$$\forall x \in (0,+\infty)$$