Norma euclidiana. Distanță euclidiană

V – spațiu vectorial real.

Se numește **normă** pe V o funcție $\|\cdot\|: V \to \mathbb{R}_+$ care îndeplinește condițiile:

1. $\|\bar{v}\| \geq 0, \ \forall \bar{v} \in V$ și $\|\bar{v}\| = 0 \Leftrightarrow \bar{v} = \bar{0}$ (pozitivitate)
2. $\|\alpha \cdot \bar{v}\| = |\alpha| \cdot \|\bar{v}\|, \ \forall \alpha \in \mathbb{R}, \bar{v} \in V$ (omogenitate)
3. $\|\bar{v} + \bar{w}\| \leq \|\bar{v}\| + \|\bar{w}\|, \ \forall \bar{v}, \bar{w} \in V$ (ineg. Δ)

$(V, \|\cdot\|)$ – spațiu vectorial normat

V – sp. vect. euclidian real

Se numește **normă euclidiană** funcția $\|\cdot\|: V \to \mathbb{R}_+$

$$\boxed{\|\bar{v}\| = \sqrt{\langle \bar{v}, \bar{v} \rangle}, \ \forall \bar{v} \in V}$$

Dacă $\bar{v} \in \mathbb{R}^n$, $\bar{v} = (\bar{v}_1, \bar{v}_2, \dots, \bar{v}_n)$

atunci $\|\bar{v}\| = \sqrt{\langle \bar{v}, \bar{v} \rangle} = \sqrt{v_1^2 + v_2^2 + \dots + v_n^2}$

$$\bar{e} = \frac{\bar{v}}{\|\bar{v}\|}, \text{ cu } \|\bar{e}\| = 1$$ – **versorul** asociat lui v

Fie $\bar{v}_1, \bar{v}_2 \neq \bar{0}$. Se numește **unghiul** dintre vectorii $\bar{v}_1$ și $\bar{v}_2$ numărul $\alpha \in [0, \pi]$, definit astfel:

$$\boxed{\cos \alpha = \frac{\langle \bar{v}_1, \bar{v}_2 \rangle}{\|\bar{v}_1\| \cdot \|\bar{v}_2\|}}$$

Fie $M \neq \phi$. Se numește distanță metrică pe M o funcție $d(\cdot,\cdot): M \times M \to \mathbb{R}_+$ care îndeplinește condițiile:

1. $d(\bar{u},\bar{v}) \geq 0, \ \forall \bar{u},\bar{v} \in M$ și $d(\bar{u},\bar{v}) = 0 \Leftrightarrow \bar{u} = \bar{v}$
2. $d(\bar{u},\bar{v}) = d(\bar{v},\bar{u}), \ \forall \bar{v},\bar{u} \in M$
3. $d(\bar{u},\bar{v}) \leq d(\bar{u},\bar{w}) + d(\bar{w},\bar{v}), \ \forall \bar{u},\bar{v},\bar{w} \in M$

(M, d) - spațiu metric.

V - sp. vectorial normat
$d(\bar{u},\bar{v}) = \|\bar{u}-\bar{v}\|$

În $\mathbb{R}^n$, $\left.\begin{array}{l}\bar{u} = (\bar{u}_1, \bar{u}_2, \dots, \bar{u}_n) \\ \bar{v} = (\bar{v}_1, \bar{v}_2, \dots, \bar{v}_n)\end{array}\right| \Rightarrow d(\bar{u},\bar{v}) = \|\bar{u}-\bar{v}\| =$

$= \sqrt{(\bar{u}_1-\bar{v}_1)^2 + \dots + (u_n - v_n)^2}$

Dacă norma este euclidiană at. dist este euclidiană.

$\boxed{\vec{v} \perp \vec{w} \Leftrightarrow \langle \bar{v}, \bar{w} \rangle = 0}$ - vectori ortogonali

Un sistem de vectori este ortonormat dacă:
- vectorii sunt ortogonali
- norma vectorilor este 1

Fie $B = \{e_1, e_2, \dots, e_n\} \subset V_n$

B - bază ortonormată $\Rightarrow \langle e_i, e_j \rangle \overset{not}{=} \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \quad i,j = \overline{1,n}$

Procedeul de ortogonalizare Gram-Schmidt

Orice spațiu vectorial euclidian real finit dimensional admite o bază ortonormată.

V_n, $\Rightarrow$ oricărei baze $B = \{\bar{u}_1, \bar{u}_2, \dots, \bar{u}_n\} \subset V_n$ i se asociază o bază ortonormată $B' = \{\bar{e}_1, \bar{e}_2, \dots, \bar{e}_n\}$, unde:

$\bar{e}_1 = \frac{\bar{v}_1}{\|\bar{v}_1\|}, \quad \bar{e}_2 = \frac{\bar{v}_2}{\|\bar{v}_2\|}, \dots, \bar{e}_n = \frac{\bar{v}_n}{\|\bar{v}_n\|}$ iar

$\bar{v}_1 = \bar{u}_1$

$$\bar{v}_2 = \bar{u}_2 - \frac{\langle \bar{u}_2, \bar{v}_1 \rangle}{\langle \bar{v}_1, \bar{v}_1 \rangle} \bar{v}_1$$

$$\bar{v}_3 = \bar{u}_3 - \frac{\langle \bar{u}_3, \bar{v}_1 \rangle}{\langle \bar{v}_1, \bar{v}_1 \rangle} \bar{v}_1 - \frac{\langle \bar{u}_3, \bar{v}_2 \rangle}{\langle \bar{v}_2, \bar{v}_2 \rangle} \cdot \bar{v}_2$$

etc.

Ex. 1 Folosind procedeul Gram-Schmidt să se ortonormeze baza $B = \{\bar{u}_1, \bar{u}_2, \bar{u}_3\} \subset \mathbb{R}^3$, unde $\bar{u}_1 = (1,1,0)$, $\bar{u}_2 = (1,0,1)$, $\bar{u}_3 = (0,0,-1)$

Notăm cu $\bar{v}_1, \bar{v}_2, \bar{v}_3$ vectori ortogonali prin G-S

$\bar{e}_1, \hat{e}_2, \bar{e}_3$ - reprezintă vectorii $\bar{v}_1, \bar{v}_2, \bar{v}_3$ aduși la lungimea $\underline{1}$.

$B' = \{\bar{e}_1, \bar{e}_2, \bar{e}_3\}$

$\bar{v}_1 = \bar{u}_1 = (1,1,0)$

$$\bar{v}_2 = \bar{u}_2 - \frac{\langle \bar{u}_2, \bar{v}_1 \rangle}{\langle \bar{v}_1, \bar{v}_1 \rangle} \cdot \bar{v}_1$$

$$\left.\begin{array}{l} \langle \bar{u}_2, \bar{v}_1 \rangle = \langle (1,0,1), (1,1,0) \rangle = 1 + 0 + 0 = 1 \\ \langle \bar{v}_1, \bar{v}_1 \rangle = \langle (1,1,0), (1,1,0) \rangle = 1 + 1 + 0 = 2 \end{array}\right| \Rightarrow$$

$$\Rightarrow \bar{v}_2 = (1,0,1) - \frac{1}{2}(1,1,0) = \left(1 - \frac{1}{2}, 0 - \frac{1}{2}, 1 - 0\right) = \left(\frac{1}{2}, \frac{-1}{2}, 1\right)$$

$$\bar{v}_3 = \bar{u}_3 - \frac{\langle \bar{u}_3, \bar{v}_1 \rangle}{\langle \bar{v}_1, \bar{v}_1 \rangle} \cdot \bar{v}_1 - \frac{\langle \bar{u}_3, \bar{v}_2 \rangle}{\langle \bar{v}_2, \bar{v}_2 \rangle} \cdot \bar{v}_2$$

$\langle \bar{u}_3, \bar{v}_1 \rangle = \langle (0,0,-1), (1,1,0) \rangle = 0 + 0 + 0 = 0$

$\langle \bar{v}_1, \bar{v}_1 \rangle = 2$

$\langle \bar{u}_3, \bar{v}_2 \rangle = \langle (0,0,-1), \left(\frac{1}{2}, -\frac{1}{2}, 1\right) \rangle = 0 - 0 - 1 = -1.$

$\langle \bar{v}_2, \bar{v}_2 \rangle = \langle \left(\frac{1}{2}, -\frac{1}{2}, 1\right), \left(\frac{1}{2}, -\frac{1}{2}, 1\right) \rangle = \frac{1}{4} + \frac{1}{4} + \frac{4}{4} = \frac{6}{4} = \frac{3}{2}$

$$\Rightarrow \bar{v}_3 = (0,0,-1) - 0 - \frac{-1}{\frac{3}{2}} \cdot \left(\frac{1}{2}, -\frac{1}{2}, 1\right) \Rightarrow$$

$\Rightarrow \bar{v}_3 = (0,0,-1) + \frac{2}{3}\left(\frac{1}{2}, -\frac{1}{2}, 1\right) = (0,0,-1) + \left(\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}\right) \Rightarrow$

$\Rightarrow \bar{v}_3 = \left(\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}\right)$

Verificăm dacă vectorii $\bar{v}_1, \bar{v}_2, \bar{v}_3$ sunt ortogonali doi câte doi $\bar{v}_1 = (1,1,0)$, $\bar{v}_2 = \left(\frac{1}{2}, -\frac{1}{2}, 1\right)$, $\bar{v}_3 = \left(\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}\right)$

$\langle \bar{v}_1, \bar{v}_2 \rangle = \langle (1,1,0), \left(\frac{1}{2}, -\frac{1}{2}, 1\right) \rangle = \frac{1}{2} - \frac{1}{2} = 0$

$\langle \bar{v}_1, \bar{v}_3 \rangle = \langle (1,1,0), \left(\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}\right) \rangle = \frac{1}{3} - \frac{1}{3} = 0$

$\langle \bar{v}_2, \bar{v}_3 \rangle = \langle \left(\frac{1}{2}, -\frac{1}{2}, 1\right), \left(\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}\right) \rangle = \underbrace{\frac{1}{6} + \frac{1}{6}}_{\frac{2}{6}} - \frac{1}{3} = 0$

Normalizăm vectorii

$\bar{e}_1 = \frac{\bar{v}_1}{\|\bar{v}_1\|}$, $\|\bar{v}_1\| = \sqrt{1^2 + 1^2 + 0^2} = \sqrt{2} \Rightarrow \bar{e}_1 = \frac{1}{\sqrt{2}}(1,1,0) \Rightarrow$

$\Rightarrow e_1 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$

$\bar{e}_2 = \frac{\bar{v}_2}{\|\bar{v}_2\|}$, $\|\bar{v}_2\| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 + 1^2} = \sqrt{\frac{1}{4} + \frac{1}{4} + 1} = \sqrt{\frac{6}{4}} = \frac{\sqrt{6}}{2}$

$\Rightarrow e_2 = \frac{1}{\frac{\sqrt{6}}{2}}\left(\frac{1}{2}, -\frac{1}{2}, 1\right) = \frac{2}{\sqrt{6}}\left(\frac{1}{2}, -\frac{1}{2}, 1\right) = \left(\frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)$

$\bar{e}_3 = \frac{\bar{v}_3}{\|\bar{v}_3\|}$, $\|\bar{v}_3\| = \sqrt{\left(\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2} = \sqrt{\frac{1}{9} + \frac{1}{9} + \frac{1}{9}} = \sqrt{\frac{3}{9}} = \frac{\sqrt{3}}{3}$

$\Rightarrow e_3 = \frac{3}{\sqrt{3}}\left(\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}\right) = \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$

Baza ortonormată obținută prin procedeul GS este

$B = \left\{ e_1 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), e_2 = \left(\frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right), e_3 = \left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right) \right\}$

Exerciții propuse:

(1) $B = \{ \bar{u}_1 = (1,0,-1), \bar{u}_2 = (0,1,1), \bar{u}_3 = (0,-1,0) \} \subset \mathbb{R}^3$

Să se ortonormeze B folosind procedeul G-S

(2) $B = \{ \bar{u}_1 = (1,1,0,0), \bar{u}_2 = (1,0,1,0), \bar{u}_3 = (1,0,0,1), \bar{u}_4 = (0,1,1,1) \} \subset \mathbb{R}^4$

Ex2

Fie $\langle f, g\rangle = \int_0^2 f(t) g(t)\,dt$, $f, g \in C^0[0,2]$

a) Verificați dacă $\langle f, g\rangle$ este produs scalar

b) Folosind G-S să se ortogonalizeze și apoi să se ortonormeze bazele:

$$B = \{1, t-2, t^3-3t\}$$

$$B' = \{1, e^x, e^{-x}\}$$

a) 1. **pozitivitatea**: $\boxed{\langle f, f\rangle \geq 0, \forall f \in C^0[0,2] \text{ și } \langle f, f\rangle = 0 \Leftrightarrow f = 0}$

$$\langle f, f\rangle = \int_0^2 f(t) \cdot f(t)\,dt = \int_0^2 \underbrace{f^2(t)}_{\geq 0}\,dt \geq 0.$$

$$\langle f, f\rangle = 0 \Rightarrow \int_0^2 f^2(t)\,dt = 0 \Rightarrow f^2(t) = 0 \Rightarrow f(t) = 0.$$

2. **simetria**: $\boxed{\langle f, g\rangle = \langle g, f\rangle, \forall f, g \in C^0[0,2]}$

$$\langle f, g\rangle = \int_0^2 f(t) g(t)\,dt = \int_0^2 g(t) \cdot f(t)\,dt = \langle g, f\rangle$$

3. **liniaritatea**: $\boxed{\langle \alpha f + \beta g, h\rangle = \alpha\langle f, h\rangle + \beta\langle g, h\rangle}$

$$\langle \alpha f + \beta g, h\rangle = \int_0^2 [\alpha f(t) + \beta g(t)] \cdot h(t)\,dt$$

$$= \int_0^2 (\alpha f(t) \cdot h(t) + \beta g(t) \cdot h(t))\,dt$$

$$= \alpha \int_0^2 f(t) \cdot h(t)\,dt + \beta \int_0^2 g(t) \cdot h(t)\,dt$$

$$= \alpha \cdot \langle f, h\rangle + \beta \cdot \langle g, h\rangle$$

b) $B = \{\underbrace{1}_{f_1}, \underbrace{t-2}_{f_2}, \underbrace{t^3-3t}_{f_3}\}$

$f_1(t) = 1$

$f_2(t) = t-2$

$f_3(t) = t^3-3t$

Notăm cu g_1, g_2, g_3 vectorii ortogonali prin G-S
$\bar{e}_1, \hat{e}_2, \bar{e}_3$ - reprezintă vectorii $\bar{v}_1, \bar{v}_2, \bar{v}_3$ aduși la lungimea 1.

$g_1 = f_1 = 1$

$$g_2 = f_2 - \frac{\langle f_2, g_1\rangle}{\langle g_1, g_1\rangle}\cdot g_1$$

$$\langle f_2, g_1\rangle = \int_0^2 f_2(t)\cdot g_1(t)\,dt = \int_0^2 (t-2)\cdot 1\,dt = \int_0^2 (t-2)\,dt$$

$$= \int_0^2 t\,dt - 2\int_0^2 dt = \frac{t^2}{2}\Big|_0^2 - 2t\Big|_0^2 = \frac{4}{2} - 2\cdot 2 = -2$$

$$\langle g_1, g_1\rangle = \int_0^2 \underbrace{g_1(t)}_{1}\cdot \underbrace{g_1(t)}_{1}\,dt = \int_0^2 dt = t\Big|_0^2 = 2$$

$$g_2 = t-2-\left(\frac{-2}{2}\right)\cdot 1 = t-2+1 = t-1$$

$$g_3 = f_3 - \frac{\langle f_3, g_1\rangle}{\langle g_1, g_1\rangle}\cdot g_1 - \frac{\langle f_3, g_2\rangle}{\langle g_2, g_2\rangle}\cdot g_2$$

$$\langle f_3, g_1\rangle = \int_0^2 \underbrace{f_3(t)}_{t^3-3t}\cdot \underbrace{g_1(t)}_{1}\,dt = \int_0^2 (t^3-3t)\,dt = \int_0^2 t^3\,dt -$$

$$-3\int_0^2 t\,dt = \frac{t^4}{4}\Big|_0^2 - 3\cdot\frac{t^2}{2}\Big|_0^2 = \frac{2^4}{4} - 3\cdot\frac{2^2}{2} = 4-6 = -2$$

$$\langle f_3, g_2\rangle = \int_0^2 \underbrace{f_3(t)}_{t^3-3t}\cdot \underbrace{g_2(t)}_{t-1}\,dt = \int_0^2 (t^3-3t)(t-1)\,dt =$$

$$= \int_0^2 (t^4-3t^2-t^3+3t)\,dt = \frac{t^5}{5}\Big|_0^2 - \cancel{3}\cdot\frac{t^3}{\cancel{3}}\Big|_0^2 - \frac{t^4}{4}\Big|_0^2 + 3\frac{t^2}{2}\Big|_0^2$$

$$= \frac{2^5}{5} - 2^3 - 4 + 6 = \frac{32}{5} - 8 - 4 + 6 = \frac{32}{5} - \overset{\frac{5}{}}{6} = \frac{2}{5}$$

$$\langle g_2, g_2\rangle = \int_0^2 g_2(t)\cdot g_2(t)\,dt = \int_0^2 (t-1)(t-1)\,dt =$$

$$= \int_0^2 (t^2-2t+1)\,dt = \frac{t^3}{3}\Big|_0^2 - 2\frac{t^2}{2}\Big|_0^2 + t\Big|_0^2 = \frac{8}{3} - 4 + 2 =$$

$$= \frac{8}{3} \overset{3}{-2} = \frac{2}{3}$$

$$g_3 = t^3 - 3t - \frac{-2}{2} \cdot 1 - \frac{\frac{2}{5}}{\frac{2}{3}} \cdot (t-1) = t^3 - 3t + 1 - \cancel{\frac{2}{5}} \cdot \frac{3}{\cancel{2}} (t-1) =$$

$$= \overset{5}{t^3} \overset{5}{-3t} \overset{5}{+1} - \frac{3}{5}(t-1) = \frac{5t^3 - 15t + 5 - 3t + 3}{5} =$$

$$g_3 = \frac{5t^3 - 18t + 8}{5}$$

$$\langle g_1, g_2 \rangle = \int_0^2 \underbrace{g_1(t)}_{1} \cdot \underbrace{g_2(t)}_{t-1} dt = \int_0^2 (t-1)\, dt = \left(\frac{t^2}{2} - t\right)\Big|_0^2 =$$

$$= 2 - 2 = 0 \implies g_1 \perp g_2$$

$$\langle g_1, g_3 \rangle = \int_0^2 \underbrace{g_1(t)}_{1} \cdot \underbrace{g_3(t)}_{\frac{5t^3 - 18t + 8}{5}} dt = \int_0^2 \frac{5t^3 - 18t + 8}{5} dt =$$

$$= \frac{\cancel{5}}{\cancel{5}} \int_0^2 t^3\, dt - \frac{18}{5} \int_0^2 t\, dt + \frac{8}{5} \int_0^2 dt =$$

$$= \frac{t^4}{4}\Big|_0^2 - \frac{\overset{9}{\cancel{18}}}{5} \frac{t^2}{\cancel{2}}\Big|_0^2 + \frac{8}{5} t\Big|_0^2 = \frac{2^4}{4} - \frac{9}{5} \cdot 2^2 + \frac{8}{5} \cdot 2 =$$

$$= 4 - \frac{36}{5} + \frac{16}{5} = 4 - \frac{20}{5} = 4 - 4 = 0 \implies g_1 \perp g_3$$

$$\langle g_2, g_3 \rangle = \int_0^2 g_2(t) \cdot g_3(t)\, dt = \int_0^2 (t-1) \cdot \frac{5t^3 - 18t + 8}{5} dt =$$

$$= \frac{1}{5} \int_0^2 (5t^4 - 5t^3 - 18t^2 + 18t + 8t - 8)\, dt =$$

$$= \frac{1}{5} \int_0^2 (5t^4 - 5t^3 - 18t^2 + 26t - 8)\, dt =$$

$$= \frac{5}{5} \int_0^2 t^4\, dt - \frac{5}{5} \int_0^2 t^3\, dt - \frac{18}{5} \int_0^2 t^2\, dt + \frac{26}{5} \int_0^2 t\, dt - \frac{8}{5} \int_0^2 dt =$$

$$= \frac{t^5}{5}\Big|_0^2 - \frac{t^4}{4}\Big|_0^2 - \frac{\overset{6}{\cancel{18}}}{5} \cdot \frac{t^3}{\cancel{3}}\Big|_0^2 + \frac{\overset{13}{\cancel{26}}}{5} \cdot \frac{t^2}{\cancel{2}}\Big|_0^2 - \frac{8}{5} t\Big|_0^2 =$$

$$= \frac{32}{5} - 4 - \frac{6}{5} \cdot 2^3 + \frac{52}{5} - \frac{8}{5} \cdot 2 = \frac{32}{5} \overset{5}{-4} - \frac{48}{5} + \frac{52}{5} - \frac{16}{5} =$$

$$= \frac{32-20-48+52-16}{5} = \frac{12+4-16}{5} = 0. \Rightarrow g_2 \perp g_3$$

$\Rightarrow g_1, g_2, g_3$ ortogonali 2 côte 2.

$$B_{ort} = \{\bar{e}_1, \bar{e}_2, \bar{e}_3\}, \quad \bar{e}_1 = \frac{g_1}{\|g_1\|}, \quad \bar{e}_2 = \frac{g_2}{\|g_2\|}, \quad e_3 = \frac{g_3}{\|g_3\|}$$

$$\|g_1\| = \sqrt{\langle g_1, g_1\rangle} = \sqrt{\int_0^2 \underbrace{g_1(t)}_{1}\cdot\underbrace{g_1(t)}_{1}} = \sqrt{\int_0^2 dt} =$$

$$= \sqrt{t\Big|_0^2} = \sqrt{2}$$

$$\|g_2\| = \sqrt{\langle g_2, g_2\rangle} = \sqrt{\frac{2}{3}}$$

$$\langle g_3, g_3\rangle = \int_0^2 g_3(t)\cdot g_3(t)\,dt = \int_0^2 \left(\frac{5t^3-18t+8}{5}\right)^2 dt =$$

$$= \frac{1}{25}\int_0^2 \left(25t^6 + 18^2t^2 + 8^2 - 2\cdot 5t^3\cdot 18t + 2\cdot 5t^3\cdot 8 - 2\cdot 18t\cdot 8\right)dt$$

$$= \frac{1}{25}\int_0^2 \left(25t^6 + 324t^2 + 64 - 180t^4 + 80t^3 - 288t\right)dt$$

$$= \frac{t^7}{5}\Big|_0^2 + \frac{\cancel{324}^{108}}{25}\cdot\frac{t^3}{\cancel{3}}\Big|_0^2 + \frac{64t}{25}\Big|_0^2 - \frac{\cancel{180}^{36}}{25}\cdot\frac{t^5}{\cancel{5}}\Big|_0^2 + \frac{\cancel{80}^{20}}{25}\cdot\frac{t^4}{\cancel{4}}\Big|_0^2 - \frac{\cancel{288}^{144}}{25}\cdot\frac{t^2}{\cancel{2}}\Big|_0^2$$

$$= \frac{128}{5} + \frac{108}{25}\cdot 8 + \frac{128}{25} - \frac{36\cdot 32}{25} + \frac{320}{25} - \frac{144\cdot 4}{25} =$$

$$= {}^{5}\frac{128}{5} + \frac{864}{25} + \frac{128-1152+320-576}{25} =$$

$$= \frac{640+864-1024-256}{25} = \frac{224}{25}$$

$$\|g_3\| = \sqrt{\langle g_3, g_3\rangle} = \sqrt{\frac{224}{25}} = \frac{\sqrt{224}}{5} = \frac{4\sqrt{14}}{5}$$

$$\bar{e}_1 = \frac{1}{\sqrt{2}}, \quad \bar{e}_2 = \frac{t-1}{\sqrt{\frac{2}{3}}}, \quad \bar{e}_3 = \frac{\frac{5t^3-18t+8}{5}}{\frac{4\sqrt{14}}{5}}$$

$$B_{ort} = \{\bar{e}_1, \bar{e}_2, \bar{e}_3\}$$