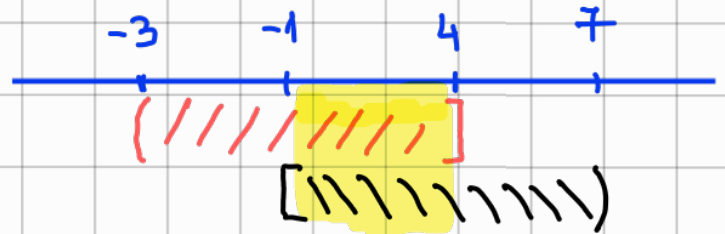


Intervale de numere reale. Exerciții rezolvate

① Efectuați următoarele intersecții de intervale.

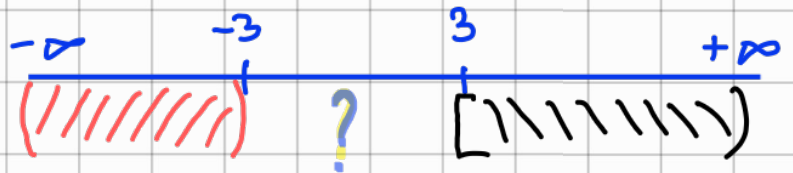
a) $(-3, 4] \cap [-1, 7)$

R: $[-1, 4]$



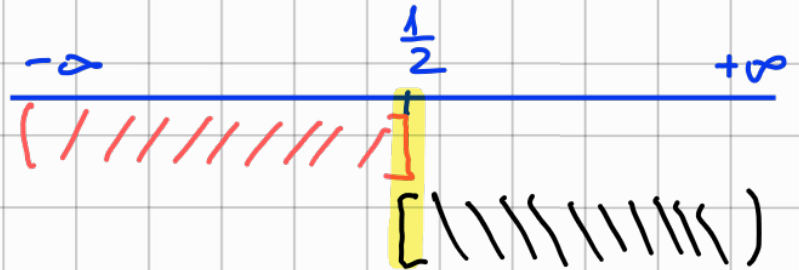
b) $(-\infty, -3) \cap [3, \infty)$

R: $\emptyset$



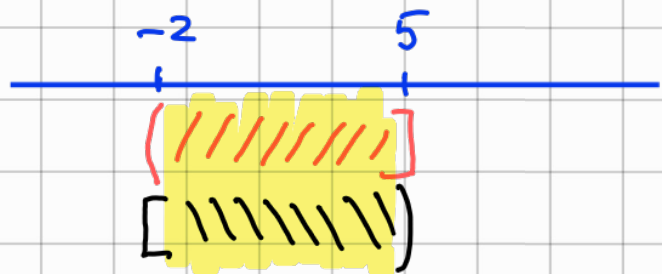
c) $(-\infty, \frac{1}{2}] \cap [\frac{1}{2}, +\infty)$

R: $\{\frac{1}{2}\}$



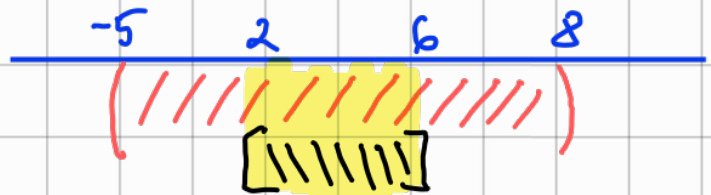
d) $[-2, 5] \cap [-2, 5)$

R: $(-2, 5)$



e) $(-5, 8) \cap [2, 6]$

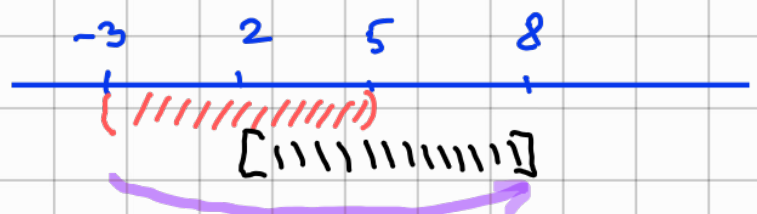
R: $[2, 6]$



② Efectuați următoarele reuniuni de intervale.

a) $(-3, 5) \cup [2, 8]$

R: $(-3, 8]$



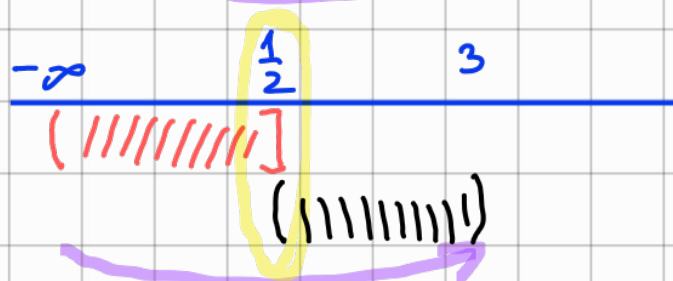
b) $[-5, 10] \cup (2, 7)$

$R: [-5, 10]$



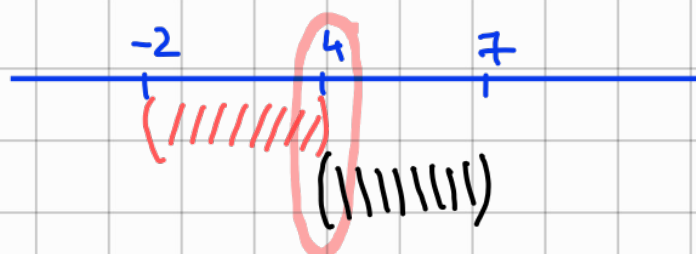
c) $(-\infty, \frac{1}{2}] \cup (\frac{1}{2}, 3)$

$R: (-\infty, 3)$



d) $(-2, 4) \cup (4, 7)$

$R: (-2, 4) \cup (4, 7)$



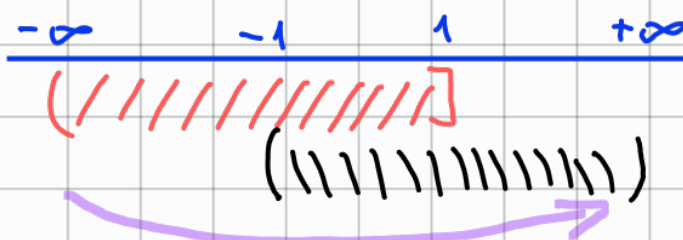
e) $[-7, -3] \cup (2, 6)$

$R: [-7, -3] \cup (2, 6)$



f) $(-\infty, 1] \cup (-1, +\infty)$

$R: \mathbb{R}$



g) $(-\infty, 2) \cup (2, +\infty)$

$R: \mathbb{R} \setminus \{2\}$



③ Scrieți următoarele mulțimi sub formă de intervale sau reuniune de intervale.

a) $A = \{x \in \mathbb{R} \mid -1 < x-3 \leq 2\}$

$-1 < x-3 \leq 2 \quad | +3$

$2 < x \leq 5$

$x \in (2, 5] \Rightarrow A = (2, 5]$

b) $B = \{x \in \mathbb{R} \mid -2 \leq \frac{2x-1}{3} < 1\}$

$$-2 \leq \frac{2x-1}{3} < 1 \quad | \cdot 3$$

$$-6 \leq 2x-1 < 3 \quad | +1$$

$$-5 \leq 2x < 4 \quad | :2$$

$$-\frac{5}{2} \leq x < 2 \Rightarrow x \in \left[-\frac{5}{2}, 2\right) \Rightarrow B = \left[-\frac{5}{2}, 2\right)$$

c) $C = \{x \in \mathbb{R} \mid |2x-1| \leq 3\}$ $|x| \leq a \Rightarrow x \in [-a, a]$

$$|2x-1| \leq 3 \Rightarrow (2x-1) \in [-3, 3] \quad | +1$$

$$2x \in [-2, 4] \quad | :2$$

$$x \in [-1, 2] \Rightarrow C = [-1, 2]$$

d) $D = \{x \in \mathbb{R} \mid |2-3x| > 1\}$ $|x| > a \Rightarrow x \in (-\infty, -a) \cup (a, +\infty)$

$$|2-3x| > 1 \Rightarrow (2-3x) \in (-\infty, -1) \cup (1, +\infty) \quad | -2$$

$$(-3x) \in (-\infty, -3) \cup (-1, +\infty) \quad | \cdot (-1)$$

$$3x \in (-\infty, 1) \cup (3, +\infty) \quad | :3$$

$$x \in (-\infty, \frac{1}{3}) \cup (1, +\infty)$$

④ Fie multimile $A = \{x \in \mathbb{R} \mid \left|\frac{1-2x}{3}\right| \leq 2\}$

$$B = \{x \in \mathbb{R} \mid \frac{x-1}{2} \in \left(-\frac{1}{2}, \frac{5}{3}\right)\}$$

a) Scrieti multimile A si B ca intervale

b) Calculati $A \cap B$, $A \cup B$, $A \setminus B$, $B \setminus A$.

a) $\left|\frac{1-2x}{3}\right| \leq 2 \Rightarrow \frac{1-2x}{3} \in [-2, 2] \quad | \cdot 3$

$$(1-2x) \in [-6, 6] \quad | -1$$

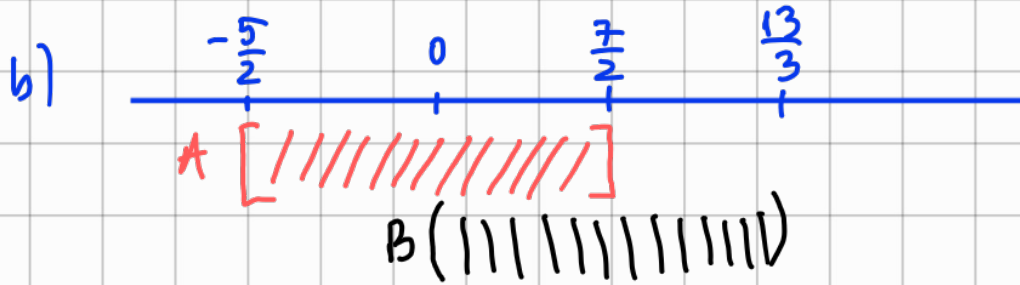
$$(-2x) \in [-7, 5] \quad | \cdot (-1)$$

$$2x \in [-5, 7] \quad | :2 \Rightarrow$$

$$x \in \left[-\frac{5}{2}, \frac{7}{2}\right] \Rightarrow A = \left[-\frac{5}{2}, \frac{7}{2}\right]$$

$$\frac{x-1}{2} \in \left(-\frac{1}{2}, \frac{5}{3}\right) \quad | \cdot 2 \Rightarrow (x-1) \in \left(-1, \frac{10}{3}\right) \quad | +1$$

$$x \in \left(0, \frac{13}{3}\right) \Rightarrow B = \left(0, \frac{13}{3}\right)$$



$$A \cap B = \left(0, \frac{7}{2}\right]$$

$A \setminus B$ - ce este în A și nu este în B

$$A \cup B = \left[-\frac{5}{2}, \frac{13}{3}\right)$$

$$A \setminus B = \left[-\frac{5}{2}, 0\right]$$

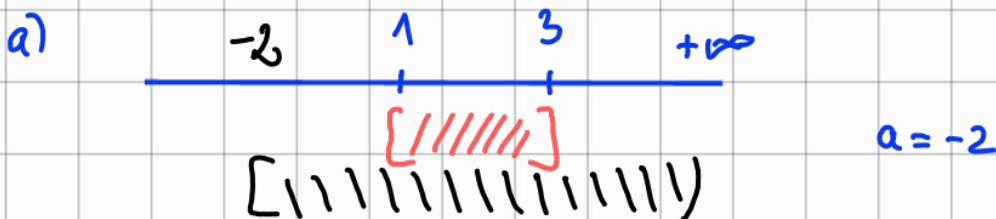
$$B \setminus A = \left(\frac{7}{2}, \frac{13}{3}\right)$$

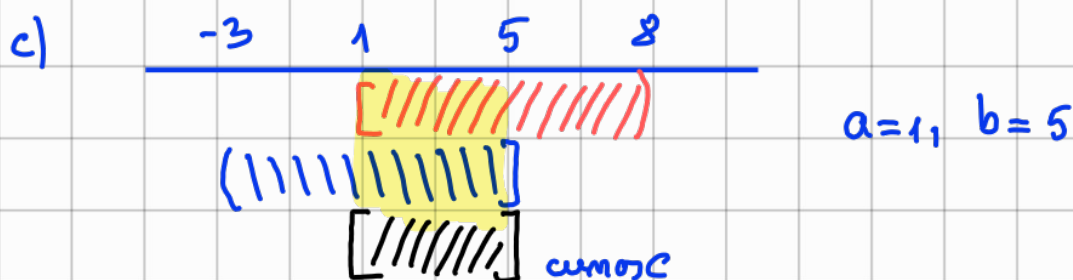
5) Determinați numerele reale a și b , $a < b$, știind că:

a) $[a, +\infty) \cup [1, 3] = [-2, +\infty)$

b) $[a, 4] \cup (-2, b) = [-5, 7)$

c) $[a, 8) \cap (-3, b] = [1, 5]$





6) Fie $a = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{9 \cdot 10}$

Verificati dacă $a \in \left(\frac{1}{3}; 0,91\right]$

$$\frac{1}{1 \cdot 2} = \frac{2-1}{1 \cdot 2} = \frac{2}{1 \cdot 2} - \frac{1}{1 \cdot 2} = 1 - \frac{1}{2}$$

$$\frac{1}{2 \cdot 3} = \frac{3-2}{2 \cdot 3} = \frac{3}{2 \cdot 3} - \frac{2}{2 \cdot 3} = \frac{1}{2} - \frac{1}{3}$$

$$\frac{1}{9 \cdot 10} = \frac{10-9}{9 \cdot 10} = \frac{10}{9 \cdot 10} - \frac{9}{9 \cdot 10} = \frac{1}{9} - \frac{1}{10}$$

$$a = 1 - \frac{1}{10} = \frac{10-1}{10} = \frac{9}{10} = 0,9$$

$$\Rightarrow a \in \left(\frac{1}{3}; 0,91\right]$$

(+)